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STABILITY PROPERTY OF A PREDATOR-PREY SYSTEM WITH A CONSTANT PROPORTION OF PREY REFUGE AND STAGE-STRUCTURE FOR PREY SPECIES*†

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Abstract

A predator-prey system with a constant proportion of prey refuge and stage-structure for prey species is proposed and studied in this paper. A set of conditions for the permanence of the system is obtained. The local stability of the system is discussed by the sign of eigenvalues. Furthermore, by using the iterative method, some suitable sufficient conditions for the global attractivity of the interior equilibrium is obtained. Our study shows that the constant proportion of prey refuge could lead to more complicate dynamic behaviors. Numerical simulations are also presented to illustrate the feasibility of the main results.

Keywords predator-prey; stage-structure; refuge; global stability

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1 Introduction

Since the pioneer work of Aiello and Freedman [1] on the single species stage-structured models, many scholars had done excellent works on the stage-structure population dynamics. For example, Lin, Xie and Chen [10] studied the convergences of a stage-structured predator-prey model with modified Leslie-Gower and Holling-type II schemes. Their study indicated that both the stage-structure and the death rate of the mature prey play important roles on the permanence or extinction of the system.

The existence of refuges sometimes plays an important role in the co-existence of predator and prey species. A prey refuge can be broadly defined as including any

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strategies to reduce the risk of predation, such as spatial or temporal refuges, prey aggregations, or reducing prey activity. As was pointed out by Devi [11], there are two types of refuges: refuges protecting a fixed number of prey and refuges protecting a constant proportion of prey. Chen et al. [12] proposed a Leslie-Gower predator-prey model incorporating a constant proportion of prey refuge. Their results showed that refuge leads to more complicate dynamic behaviors.

There are many works on stage-structure population dynamics [1-20], and many works on the predator prey system incorporate the prey refuge [11-14]. However, only recently, Devi [11] proposed a stage-structured predator-prey model with prey refuge. They considered the refuges protecting a fixed number of prey. Devi showed that the equilibrium value of mature prey population increases with the prey refuges, whereas the equilibrium value of predator population decreases with the prey refuges. The success of Devi [11] motivates us to consider the influence of a constant proportion of prey refuge and to propose the following model:

$$\begin{aligned}
&\dot{x}_i(t) = \alpha x_m(t) - \gamma x_i(t) - \alpha \mathrm{e}^{-\gamma\tau} x_m(t-\tau),\\
&\dot{x}_m(t) = \alpha \mathrm{e}^{-\gamma\tau} x_m(t-\tau) - \beta x_m^2(t) - \frac{(1-p)x_m(t)y(t)}{(1-p)x_m(t)+y(t)},\\
&\dot{y}(t) = k\frac{(1-p)x_m(t)y(t)}{(1-p)x_m(t)+y(t)} - dy(t), \qquad\qquad (1.1)\\
&x_m(\theta) = \phi_m(\theta) \geq 0, \quad -\tau \leq \theta < 0, \quad x_i(0) > 0, \quad x_m(0) > 0, \quad y(0) > 0,
\end{aligned}$$

where $x_i(t)$ and $x_m(t)$ represent the densities of the immature and mature prey populations, respectively. $y(t)$ is described as the density of predator population at time t.

System (1.1) satisfies the following assumptions:

(1) The per capita birth rate of the immature popular is $\alpha > 0$. The per capita death rate of the immature popular is $\gamma > 0$. The per capita death rate of the mature prey is proportional to the current mature prey population with a proportionality constant $\beta > 0$. $\tau > 0$ is the length of time from birth to maturity. $\mathrm{e}^{-\gamma\tau}$ denotes the surviving rate of immaturity to reach maturity. The term $\alpha \mathrm{e}^{-\gamma\tau} x_m(t-\tau)$ models the immature individuals who are born at time $t-\tau$ and survive and mature at time t;

(2) It is assumed that predators only feed on the mature prey. $k > 0$ is the efficiency with which predators convert consumed prey into new predators. $d > 0$ is the death rate of predators species. The mature prey using refuges are proportional to the existing population with a proportionality constant $0 < p < 1$.

For the continuity of the solutions to system (1.1), in this paper, we require

$$x_i(0) = \int_{-\tau}^{0} \alpha \mathrm{e}^{\gamma s} \phi_m(s) \mathrm{d}s.$$

Now integrating both sides of the first equation of system (1.1) in the interval $(0, t)$, we obtain that

$$x_i(t) = \int_{t-\tau}^{t} \alpha \mathrm{e}^{-\gamma(t-s)} x_m(s) \mathrm{d}s. \tag{1.2}$$

From (1.2), one could easily see that the dynamic behaviors of $x_i(t)$ is determined by $x_m(t)$. Hence, we only need to analyse the the following subsystem of system (1.1)

$$\begin{aligned}
\dot{x}_m(t) &= \alpha \mathrm{e}^{-\gamma\tau} x_m(t-\tau) - \beta x_m^2(t) - \frac{(1-p)x_m(t)y(t)}{(1-p)x_m(t)+y(t)}, \\
\dot{y}(t) &= k\frac{(1-p)x_m(t)y(t)}{(1-p)x_m(t)+y(t)} - dy(t), \\
x_m(\theta) &= \phi_m(\theta) \geq 0, \quad -\tau \leq \theta < 0, \quad x_m(0) > 0, \quad y(0) > 0.
\end{aligned} \tag{1.3}$$

2 Permanence

To investigate the persistent property of the system,we need the following lemmas.

Lemma 2.1[9] *Consider the following equation:*

$$\dot{x}(t) = ax(t-\tau) - bx(t) - cx^2(t), \tag{2.1}$$

where $a, b, c, \tau > 0$ *and* $x(t) > 0, for - \tau \leq t \leq 0.$

(1) *If* $a > b$, *then* $\lim\limits_{t\to\infty} x(t) = \frac{a-b}{c}$.

(2) *If* $a < b$, *then* $\lim\limits_{t\to\infty} x(t) = 0$.

Lemma 2.2[21]

(1) *If* $a > 0$, $b > 0$, *and* $\dot{x}(t) \geq a - bx(t)$, *when* $t \geq 0$ *and* $x(0) > 0$, *then* $\liminf\limits_{t\to\infty} x(t) \geq \frac{a}{b}$.

(2) *If* $a > 0$, $b > 0$, *and* $\dot{x}(t) \leq a - bx(t)$, *when* $t \geq 0$ *and* $x(0) > 0$, *then* $\limsup\limits_{t\to\infty} x(t) \leq \frac{a}{b}$.

Lemma 2.3[21]

(1) *If* $a > 0$, $b > 0$, *and* $\dot{x}(t) \geq x(t)\big(a - bx(t)\big)$, *when* $t \geq 0$ *and* $x(0) > 0$, *then* $\liminf\limits_{t\to\infty} x(t) \geq \frac{a}{b}$.

(2) *If* $a > 0$, $b > 0$, *and* $\dot{x}(t) \leq x(t)\big(a - bx(t)\big)$, *when* $t \geq 0$ *and* $x(0) > 0$, *then* $\limsup\limits_{t\to\infty} x(t) \leq \frac{a}{b}$.

Similar to the proof of Theorem 3.1 in [11], we can obtain:

Lemma 2.4 *Assume* $\phi(\theta) \geq 0$ *is continuous on* $\theta \in [-\tau, 0]$, $x_m(0), y(0) > 0$, *then the solutions of system* (1.3) *are positive for all* $t > 0$.

Theorem 2.1 *All solutions of system* (1.3) *are bounded on* Ω, *where*

$$\Omega=\{(x_m(t),y(t)):x_m(t)\le M_1,y(t)\le M_2\}. \tag{2.2}$$

Proof Consider the first equation of system (1.3):

$$\dot{x}_m(t)\le \alpha \mathrm{e}^{-\gamma\tau}x_m(t-\tau)-\beta x_m^2(t). \tag{2.3}$$

By Lemma 2.1 and the comparison theorem, we obtain

$$\limsup_{t\to\infty} x_m(t)\le \frac{\alpha \mathrm{e}^{-\gamma\tau}}{\beta}=M_1, \tag{2.4}$$

and so, for $\varepsilon>0$ enough small, there exists an enough T such that

$$x_m(t)<M_1+\varepsilon.$$

From the second equation of system (1.3), it follows that

$$\dot{y}(t)\le k(1-p)(M_1+\varepsilon)-dy(t). \tag{2.5}$$

Applying Lemma 2.2 to (2.5), we obtain

$$\limsup_{t\to\infty} y(t)\le \frac{k(1-p)(M_1+\varepsilon)}{d}=M_2^{\varepsilon}.$$

Setting $\varepsilon\to 0$ leads to

$$\limsup_{t\to\infty} y(t)\le \frac{k(1-p)M_1}{d}=M_2. \tag{2.6}$$

The proof of Theorem 2.1 is completed.

Theorem 2.2 *Assume that* $k>d$ *and* $\alpha \mathrm{e}^{-\gamma\tau}>1-p$, *then system* (1.3) *is permanent.*

Proof From the first equation of system (1.3), one has

$$\dot{x}_m(t)\ge \alpha \mathrm{e}^{-\gamma\tau}x_m(t-\tau)-\beta x_m^2(t)-(1-p)x_m(t). \tag{2.7}$$

By Lemma 2.1 and the comparison theorem, we can obtain that

$$\liminf_{t\to\infty} x_m(t)\ge \frac{\alpha \mathrm{e}^{-\gamma\tau}-(1-p)}{\beta}=m_1. \tag{2.8}$$

For $\varepsilon>0$ small enough, there exists an enough large T^* such that

$$x_m(t)>m_1-\varepsilon.$$

Consider the second equation of system (1.3)

$$\dot{y}(t)\ge \frac{(k-d)(1-p)(m_1-\varepsilon)y(t)-dy^2(t)}{(1-p)(m_1-\varepsilon)+(M_2+\varepsilon)}. \tag{2.9}$$

By Lemma 2.3, we obtain

$$\liminf_{t\to\infty} y(t) \geq \frac{(k-d)(1-p)(m_1-\varepsilon)}{d} = m_2^{\varepsilon}.$$

Setting $\varepsilon \to 0$ leads to

$$\liminf_{t\to\infty} y(t) \geq \frac{(k-d)(1-p)m_1}{d} = m_2. \tag{2.10}$$

Hence, according to (2.4), (2.6), (2.8) and (2.10), we obtain that

$$\begin{aligned} &m_1 \leq \liminf_{t\to\infty} x_m(t) \leq \limsup_{t\to\infty} x_m(t) \leq M_1, \\ &m_2 \leq \liminf_{t\to\infty} y(t) \leq \limsup_{t\to\infty} y(t) \leq M_2. \end{aligned} \tag{2.11}$$

This ends the proof of Theorem 2.2.

3 Stability of the Equilibria

According to the equations of system (1.3), we can obtain two nonnegative equilibrium points:

$$A\left(\frac{\alpha \mathrm{e}^{-\gamma\tau}}{\beta}, 0\right), \quad B(x_m^*, y^*), \tag{3.1}$$

where

$$\begin{aligned} x_m^* &= \frac{\alpha \mathrm{e}^{-\gamma\tau}}{\beta} - \frac{(k-d)(1-p)}{k\beta}, \\ y^* &= \frac{(k-d)(1-p)\alpha \mathrm{e}^{-\gamma\tau}}{d\beta} - \frac{(k-d)^2(1-p)^2}{kd\beta}. \end{aligned} \tag{3.2}$$

Clearly, the positive equilibrium point B exists if

$$k > d, \quad \alpha \mathrm{e}^{-\gamma\tau} > \frac{(k-d)(1-p)}{k}. \tag{3.3}$$

Now, consider the influence of the prey refuse on the density of both prey and predator species.

Since x_m^* is a continuous function of parameter p, direct computation shows that

$$\frac{\mathrm{d}x_m^*}{\mathrm{d}p} = \frac{k-d}{k\beta} > 0, \quad p \in (0,1).$$

The above inequality shows that x_m^* is a strictly increasing function of p. That is, increasing the number of refuge can increase prey densities.

On the other hand, since y^* is a continuous function of p, direct computation shows that

$$\frac{\mathrm{d}y^*}{\mathrm{d}p} = \frac{\big(k-d\big)\big(2(k-d)(1-p) - \alpha \mathrm{e}^{-\gamma\tau}k\big)}{kd\beta}, \quad p \in (0,1).$$

(1) Assume that $2(k-d) \leq k\alpha \mathrm{e}^{-\gamma\tau}$, then

$$\frac{\mathrm{d}y^*}{\mathrm{d}p} < 0, \quad p \in (0,1).$$

Therefore, y^* is a strictly decreasing function of $p \in (0,1)$. That is increasing the number of prey refuge can decrease the predator densities.

(2) Assume that $2(k-d) > k\alpha \mathrm{e}^{-\gamma\tau}$. Let

$$f(p) = \frac{(k-d)\big(2(k-d)(1-p) - \alpha \mathrm{e}^{-\gamma\tau} k\big)}{kd\beta}.$$

$f(p) = 0$ has a unique positive solution $p^* = \frac{k\alpha \mathrm{e}^{-\gamma\tau} + 2d - 2k}{2(d-k)}$. It follows that

$$\frac{\mathrm{d}y^*}{\mathrm{d}p} > 0, \quad \text{for all } p \in (0, p^*).$$

That is, there exists a threshold $p = p^*$ such that for all $p \in (0, p^*)$, y^* is a strictly increasing function of p, otherwise y^* is a strictly decreasing function of p, for all $p \in (p^*, 1)$. Hence, when the prey refuge is large enough, increasing the number of prey refuge can decrease the predator densities. The predator species achieve their maximum densities at the threshold $p = p^*$.

Theorem 3.1 *Assume that $k < d$, then the equilibrium point A is locally asymptotically stable.*

Proof The variational matrix of system (1.3) at the equilibrium point A is

$$V(A) = \begin{pmatrix} \alpha \mathrm{e}^{-(\gamma+\lambda)\tau} - 2\alpha \mathrm{e}^{-\gamma\tau} & -1 \\ 0 & k-d \end{pmatrix}.$$

We can obtain the following characteristic equation at the equilibrium point A:

$$(\lambda - \alpha \mathrm{e}^{-(\gamma+\lambda)\tau} + 2\alpha \mathrm{e}^{-\gamma\tau})(\lambda - k + d) = 0. \tag{3.4}$$

One solution of characteristic equation at point A is $\lambda = k - d$, and other solutions are given by

$$\lambda - \alpha \mathrm{e}^{-(\gamma+\lambda)\tau} + 2\alpha \mathrm{e}^{-\gamma\tau} = 0, \tag{3.5}$$

which implies $\mathrm{Re}\lambda < 0$. We prove it by contradiction. Suppose that $\mathrm{Re}\lambda \geq 0$, then we can obtain

$$\mathrm{Re}\lambda = \alpha \mathrm{e}^{-\gamma\tau} \mathrm{e}^{-\mathrm{Re}(\lambda)\tau} \cos(\mathrm{Im}\lambda)\tau - 2\alpha \mathrm{e}^{-\gamma\tau} \leq \alpha \mathrm{e}^{-\gamma\tau} - 2\alpha \mathrm{e}^{-\gamma\tau} < 0. \tag{3.6}$$

It is a contradiction, hence, $\mathrm{Re}\lambda < 0$. The above analysis shows that A is unstable if $k > d$ and A is locally asymptotically stable if $k < d$.

To investigate the stability property of the equilibrium B, we need the following lemma.

Lemma 3.2[20] *The necessary and sufficient conditions for the equilibrium point B to be asymptotically stable for $\tau \geq 0$ is the following conditions:*

(1) *The real parts of all the eigenvalue are negative in $F(\lambda, 0) = 0$.*

(2) *For all real b and $\tau \geq 0$, $F(ib, \tau) \neq 0$, where $i = \sqrt[2]{-1}$.*

Theorem 3.1 *Assume that $k > d$ and $k \geq 2(1-p)$, then the equilibrium point B is locally asymptotically stable.*

Proof The variational matrix of system (1.3) at the equilibrium point B is

$$V(B) = \begin{pmatrix} \alpha \mathrm{e}^{-(\gamma+\lambda)\tau} - 2\alpha \mathrm{e}^{-\gamma\tau} - \dfrac{(k-d)(k+d)(1-p)}{k^2} & -\dfrac{d^2}{k^2} \\ \dfrac{(k-d)(1-p)}{k} & \dfrac{d^2-kd}{k} \end{pmatrix}$$

The characteristic equation at the equilibrium point B is

$$F(\lambda, \tau) = \lambda^2 + \lambda P_1(\tau) + P_0(\tau) + (\lambda Q_1(\tau) + Q_0(\tau))\mathrm{e}^{-\lambda\tau} = 0, \tag{3.7}$$

where

$$\begin{aligned} P_1(\tau) &= 2\alpha \mathrm{e}^{-\gamma\tau} + \frac{(k-d)(k+d)(1-p)}{k^2} + \frac{kd-d^2}{k}, \\ P_0(\tau) &= 2\alpha \mathrm{e}^{-\gamma\tau} \frac{kd-d^2}{k} + \frac{d(k-d)^2(k+2d)(1-p)}{k^3}, \\ Q_1(\tau) &= -\alpha \mathrm{e}^{-\gamma\tau}, \quad Q_0(\tau) = -\alpha \mathrm{e}^{-\gamma\tau} \frac{kd-d^2}{k}. \end{aligned}$$

According to Lemma 3.2, we need to take two steps to prove Theorem 3.1.

(1) From equation (3.7), we have

$$F(\lambda, 0) = \lambda^2 + \lambda\big(P_1(0) + Q_1(0)\big) + \big(P_0(0) + Q_0(0)\big) = 0. \tag{3.8}$$

We consider the signs of $P_1(0) + Q_1(0)$ and $P_0(0) + Q_0(0)$,

$$\begin{aligned} P_1(0) + Q_1(0) &= \frac{(k-d)d}{k} + 2\alpha + \frac{(k-d)(k+d)(1-p)}{k^2} - \alpha > 0, \\ P_0(0) + Q_0(0) &= \frac{2\alpha(k-d)d}{k} + \frac{d(k-d)^2(1-p)(k+2d)}{k^3} - \frac{\alpha(k-d)d}{k} \\ &= \frac{\alpha(k-d)d}{k} + \frac{d(k-d)^2(1-p)(k+2d)}{k^3} > 0. \end{aligned}$$

Then all the roots of equation (3.8) have negative real parts. Hence, the equilibrium point B is locally asymptotically stable at $\tau = 0$.

(2) We consider $F(ib_0, \tau_0) = 0$, for real b_0.

Firstly, when $b_0 = 0$,

$$\begin{aligned}F(0,\tau_0) &= P_0(\tau_0)+Q_0(\tau_0)\\&= 2\alpha e^{-\gamma\tau}\frac{kd-d^2}{k}+\frac{d(k-d)^2(k+2d)(1-p)}{k^3}-\alpha e^{-\gamma\tau}\frac{kd-d^2}{k}\\&= \alpha e^{-\gamma\tau}\frac{kd-d^2}{k}+\frac{d(k-d)^2(k+2d)(1-p)}{k^3}>0.\end{aligned}\tag{3.9}$$

Secondly, when $b_0\neq 0$, depending on the signs of $A(\tau_0)$ and $B(\tau_0)$, equation (3.7) does not have the positive real roots

$$\begin{aligned}&F(ib_0,\tau_0) = -b_0^2+ib_0P_1(\tau_0)+P_0(\tau)+(ib_0Q_1(\tau_0)+Q_0(\tau_0))(\cos b_0\tau_0-i\sin b_0\tau_0)=0,\\&F(ib_0,\tau_0) = F_R(ib_0,\tau_0)+iF_I(ib_0,\tau_0),\\&F_R(ib_0,\tau_0) = -b_0^2+P_0(\tau_0)+b_0Q_1(\tau_0)\sin b_0\tau_0+Q_0(\tau_0)\cos b_0\tau_0,\\&F_I(ib_0,\tau_0) = b_0P_1(\tau_0)+b_0Q_1(\tau_0)\cos b_0\tau_0-Q_0(\tau_0)\sin b_0\tau_0,\\&F(b_0,\tau_0) = F_R^2(ib_0,\tau_0)+F_I^2(ib_0,\tau_0)=0.\end{aligned}$$

We obtain

$$\begin{aligned}&F(b_0,\tau_0)=b^4+A(\tau_0)b^2+B(\tau_0)=0,\\&A(\tau_0)=-2P_0(\tau_0)+P_1^2(\tau_0)-Q_1^2(\tau_0),\\&B(\tau_0)=P_0^2(\tau_0)-Q_0^2(\tau_0).\end{aligned}\tag{3.10}$$

By calculation,

$$P_0(\tau_0)-Q_0(\tau_0)=3\alpha e^{-\gamma\tau_0}\frac{kd-d^2}{k}+\frac{d(k-d)^2(k+2d)(1-p)}{k^3}>0.$$

First, we consider the sign of $B(\tau_0)$,

$$B(\tau_0)=P_0^2(\tau_0)-Q_0^2(\tau_0)=(P_0(\tau_0)+Q_0(\tau_0))(P_0(\tau_0)-Q_0(\tau_0)).\tag{3.11}$$

From (3.9) and (3.10), $P_0(\tau_0)+Q_0(\tau_0)>0$, $P_0(\tau_0)-Q_0(\tau_0)>0$. Therefore, we can obtain $B(\tau_0)>0$.

Next, we discuss the sign of $A(\tau_0)$,

$$\begin{aligned}A(\tau_0) &= -2P_0(\tau_0)+P_1^2(\tau_0)-Q_1^2(\tau_0)\\&= -2\Big(2\alpha e^{-\gamma\tau_0}\frac{kd-d^2}{k}+\frac{d(k-d)^2(k+2d)(1-p)}{k^3}\Big)-(-\alpha e^{-\gamma\tau_0})^2\\&\quad+\Big(2\alpha e^{-\gamma\tau_0}+\frac{(k-d)(k+d)(1-p)}{k^2}+\frac{kd-d^2}{k}\Big)^2\\&= -2\Big(2\alpha e^{-\gamma\tau_0}\frac{kd-d^2}{k}\Big)-2\frac{d(k-d)^2(k+2d)(1-p)}{k^3}-\alpha^2e^{-2\gamma\tau_0}\\&\quad+\Big(\frac{(k-d)(k+d)(1-p)}{k^2}\Big)^2+\Big(2\alpha e^{-\gamma\tau_0}+\frac{kd-d^2}{k}\Big)^2\\&\quad+2\frac{(k-d)(k+d)(1-p)}{k^2}\Big(2\alpha e^{-\gamma\tau_0}+\frac{kd-d^2}{k}\Big)\end{aligned}$$

$$
\begin{aligned}
&= \frac{(k-d)^2d^2}{k^2} + 3\alpha^2\mathrm{e}^{-2\gamma\tau_0} + \frac{(k-d)^2(k+d)^2(1-p)^2}{k^4} \\
&\quad - \frac{2(k-d)^2d^2(1-p)}{k^3} + 4\alpha\mathrm{e}^{-\gamma\tau_0}\frac{(k-d)(k+d)(1-p)}{k^2} \\
&= \frac{(k-d)^2d^2}{k^3}\big(k-2(1-p)\big) + 4\alpha\mathrm{e}^{-\gamma\tau_0}\frac{(k-d)(k+d)(1-p)}{k^2} \\
&\quad +3\alpha^2\mathrm{e}^{-2\gamma\tau_0} + \frac{(k-d)^2(k+d)^2(1-p)^2}{k^4}.
\end{aligned}
\tag{3.12}
$$

By the conditions of Theorem 3.1, we have

$$
\begin{aligned}
A(\tau_0) &= -2P_0(\tau_0) + P_1^2(\tau_0) - Q_1^2(\tau_0), \\
&= \frac{(k-d)^2d^2}{k^3}\big(k-2(1-p)\big) + 4\alpha\mathrm{e}^{-\gamma\tau_0}\frac{(k-d)(k+d)(1-p)}{k^2} \\
&\quad +3\alpha^2\mathrm{e}^{-2\gamma\tau_0} + \frac{(k-d)^2(k+d)^2(1-p)^2}{k^4} > 0.
\end{aligned}
\tag{3.13}
$$

Therefore, $F(ib_0,\tau_0) \neq 0$ for any real b_0. According to Lemma 3.1, the equilibrium point B is locally asymptotically stable.

4 Global Attractivity

In this section, we study the global stability property of the equilibrium points A and B. By Theorem 4.9.1 in Kuang [18], we can conclude the following lemma.

Lemma 4.1

$$
\begin{aligned}
&\dot{v}(t) = a_4v(t-\tau) - a_3v^2(t) - \frac{a_1v(t)}{v(t)+a_2}, \\
&v(t) = \varphi(t) \geq 0, \quad t \in [-\tau, 0], \\
&v(0) > 0.
\end{aligned}
\tag{4.1}
$$

Assume that $a_2a_4 - a_1 > 0$, then system (4.1) admits a unique positive equilibrium,

$$
\lim_{t\to\infty} v(t) = v^* = \frac{a_4 - a_2a_3 + \sqrt{(a_4-a_2a_3)^2 + 4a_3(a_2a_4-a_1)}}{2a_3}, \tag{4.2}
$$

which is globally asymptotically stable.

Theorem 4.1 *Assume that $k < d$ and $\alpha\mathrm{e}^{-\gamma\tau} > 1-p$, then the equilibrium point A is globally attractive.*

Proof From the first equations of system (1.3), we obtain

$$
\dot{x}_m(t) \leq \alpha\mathrm{e}^{-\gamma\tau}x_m(t-\tau) - \beta x_m^2(t). \tag{4.3}
$$

Consider the following system

$$
\begin{aligned}
&\dot{u}(t) = \alpha\mathrm{e}^{-\gamma\tau}u(t-\tau) - \beta u(t), \quad t \geq 0, \\
&u(t) = \varphi(t), \quad -\tau \leq t \leq 0.
\end{aligned}
$$

It follows from Lemma 2.1 that

$$\lim_{t\to\infty} u(t) = \frac{\alpha e^{-\gamma\tau}}{\beta}.$$

By the comparison theorem, we have $x_m(t) \le u(t)$, $t \ge 0$. Hence, for any sufficiently small positive number ε, there exists a $T_1^* > 0$ such that

$$x_m(t) \le \frac{\alpha e^{-\gamma\tau}}{\beta} + \varepsilon, \quad t \ge T_1^*. \tag{4.4}$$

From the second equation of system (1.3), we obtain

$$\dot{y} \le (k-d)y(t), \quad t \ge T_1^*. \tag{4.5}$$

Hence

$$y(t) \le \exp\{(k-d)t\}.$$

Since $k < d$, one could easily see that $y(t) \to 0$ as $t \to \infty$. Therefore, for any sufficiently small positive number ε, there exists a T_2^* such that

$$y(t) \le \varepsilon, \quad t \ge T_2^*. \tag{4.6}$$

From the first equations of system (1.3) and (4.6), we can obtain

$$\dot{x}_m(t) \ge \alpha^{-\gamma\tau} x_m(t-\tau) - \beta x_m^2(t) - \frac{x_m(t)\varepsilon}{x_m(t) + \dfrac{\varepsilon}{1-p}}, \quad t \ge T_2^* + \tau.$$

Consider the following system

$$\begin{aligned} &\dot{u}(t) = \alpha^{-\gamma\tau} u(t-\tau) - \beta u^2(t) - \frac{u(t)\varepsilon}{u(t) + \dfrac{\varepsilon}{1-p}}, \\ &u(t) = \varphi(t), \quad T_2^* \le t \le T_2^* + \tau. \end{aligned}$$

Because of $\alpha e^{-\gamma\tau} > 1-p$, it follows from Lemma 4.1 that

$$\lim_{t\to\infty} u(t) = \frac{\alpha e^{-\gamma\tau}}{2\beta} - \frac{\varepsilon}{2(1-p)} + \frac{\sqrt{\Big(\alpha^{-\gamma\tau} - \dfrac{\beta\varepsilon}{1-p}\Big)^2 + 4\beta\Big(\dfrac{\alpha^{-\gamma\tau}}{1-p} - 1\Big)\varepsilon}}{2\beta} - \varepsilon,$$

and by the comparison theorem, we obtain $x_m(t) \ge u(t)$, $t \ge T_2^* + \tau$. Hence,

$$\begin{aligned} x_m(t) &\ge \frac{\alpha e^{-\gamma\tau}}{2\beta} - \frac{\varepsilon}{2(1-p)} + \frac{\sqrt{\Big(\alpha^{-\gamma\tau} - \dfrac{\beta\varepsilon}{1-p}\Big)^2 + 4\beta\Big(\dfrac{\alpha^{-\gamma\tau}}{1-p} - 1\Big)\varepsilon}}{2\beta} - \varepsilon \\ &\ge \frac{\alpha e^{-\gamma\tau}}{2\beta} - \frac{\varepsilon}{2(1-p)} + \frac{\alpha^{-\gamma\tau} - \dfrac{\beta\varepsilon}{1-p}}{2\beta} - \varepsilon \\ &= \frac{\alpha e^{-\gamma\tau}}{\beta} - \frac{\varepsilon}{1-p} - \varepsilon. \end{aligned} \tag{4.7}$$

It follows from (4.4) and (4.7) that

$$\lim_{t\to\infty} x_m(t) = \frac{\alpha e^{-\gamma\tau}}{\beta}, \tag{4.8}$$

$$\lim_{t\to\infty} (x_m(t), y(t)) = \Big(\frac{\alpha e^{-\gamma\tau}}{\beta}, 0\Big). \tag{4.9}$$

This proof is completed.

Theorem 4.2 *If $k < 2d$ and $\alpha e^{-\gamma\tau} > 1 - p$, then the equilibrium point B is globally attractive.*

Proof Consider the first equation of system (1.3),

$$\dot{x}_m(t) \le \alpha e^{-\gamma\tau} x_m(t-\tau) - \beta x_m^2(t). \tag{4.10}$$

Consider the following system

$$\begin{aligned} &\dot{u}(t) = \alpha e^{-\gamma\tau} u(t-\tau) - \beta u^2(t), \quad t \ge 0, \\ &u(t) = \varphi(t), \quad -\tau \le t \le 0. \end{aligned}$$

It follows Lemma 2.1 that $\lim\limits_{t\to\infty} u(t) = \alpha e^{-\gamma\tau}/\beta$. By the comparison principle, we obtain $x_m(t) \le u(t)$, $t \ge 0$. Therefore, for any sufficiently small $\varepsilon > 0$, there exists a $T_1 > 0$ such that

$$x_m(t) \le \frac{\alpha e^{-\gamma\tau}}{\beta} + \varepsilon = \bar{u}_1, \quad t \ge T_1. \tag{4.11}$$

From the second equation of system (1.3) and (4.11), we obtain

$$\dot{y}(t) \le \frac{k(1-p)\bar{u}_1 y(t)}{(1-p)\bar{u}_1 + y(t)} - dy(t), \quad t \ge T_1. \tag{4.12}$$

By using the comparison theorem of the differential equation, there exists a $T_2 > T_1$ such that

$$y(t) \le \frac{(k-d)(1-p)\bar{u}_1}{d} + \varepsilon = \bar{v}_1, \quad t \ge T_2. \tag{4.13}$$

From (4.13) and the first equation of system (1.3), we obtain

$$\dot{x}_m \ge \alpha^{-\gamma\tau} x_m(t-\tau) - \beta x_m^2(t) - \frac{x_m(t)\bar{v}_1}{x_m(t) + \dfrac{\bar{v}_1}{1-p}}, \quad t \ge T_2 + \tau. \tag{4.14}$$

Consider the following system

$$\begin{aligned} &\dot{u}(t) = \alpha^{-\gamma\tau} u(t-\tau) - \beta u^2(t) - \frac{u(t)\bar{v}_1}{u(t) + \dfrac{\bar{v}_1}{1-p}}, \quad t \ge T_2 + \tau, \\ &u(t) = \varphi(t), \quad T_2 \le t \le T_2 + \tau. \end{aligned}$$

It follows from Lemma 4.1 that

$$\lim_{t\to\infty} u(t) = \frac{\alpha e^{-\gamma\tau}}{2\beta} - \frac{\bar{v}_1}{2(1-p)} + \frac{\sqrt{\Big(\alpha^{-\gamma\tau} - \frac{\beta\bar{v}_1}{1-p}\Big)^2 + 4\beta\Big(\frac{\alpha^{-\gamma\tau}}{1-p} - 1\Big)\bar{v}_1}}{2\beta}.$$

By the comparison principle, we have $x_m(t) \geq u(t)$, $t \geq T_2 + \tau$. Hence, there exists a $T_3 > T_2 + \tau > 0$ such that

$$x_m(t) \geq \frac{\alpha e^{-\gamma\tau}}{2\beta} - \frac{\bar{v}_1}{2(1-p)} + \frac{\sqrt{\Big(\alpha^{-\gamma\tau} - \frac{\beta\bar{v}_1}{1-p}\Big)^2 + 4\beta\Big(\frac{\alpha^{-\gamma\tau}}{1-p} - 1\Big)\bar{v}_1}}{2\beta} - \varepsilon = \underline{u}_1. \tag{4.15}$$

From (4.15) and the second equation of system (1.3), we obtain

$$\dot{y}(t) \geq \frac{k(1-p)\underline{u}_1 y(t) - dy^2(t)}{(1-p)\underline{u}_1 + \bar{v}_1}, \quad t \geq T_3.$$

By the comparison principle and Lemma 2.3, there exists a $T_4 > T_3$ such that

$$y(t) \geq \frac{(k-d)(1-p)\underline{u}_1}{d} - \varepsilon = \underline{v}_1, \quad t \geq T_4. \tag{4.16}$$

By the similar arguments as above and Lemma 4.1, for any sufficiently small $\varepsilon > 0$, there exists a $T_5 > T_4 + \tau > 0$ such that

$$x_m(t) \leq \frac{\alpha e^{-\gamma\tau}}{2\beta} - \frac{\underline{v}_1}{2(1-p)} + \frac{\sqrt{\Big(\alpha^{-\gamma\tau} - \frac{\beta\underline{v}_1}{1-p}\Big)^2 + 4\beta\Big(\frac{\alpha^{-\gamma\tau}}{1-p} - 1\Big)\underline{v}_1}}{2\beta} + \frac{\varepsilon}{2} = \bar{u}_2. \tag{4.17}$$

From (4.11) and (4.17), we can have

$$\bar{u}_1 > \bar{u}_2. \tag{4.18}$$

From (4.17) and the second equation of system (1.3), we obtain

$$\dot{y}(t) \leq \frac{k(1-p)\bar{u}_2 y(t) - dy^2(t)}{(1-p)\bar{u}_2 + \underline{v}_1}, \quad t \geq T_5. \tag{4.19}$$

By the comparison theorem and Lemma 2.3, there exists a $T_6 > T_5$ such that

$$y(t) < \frac{(k-d)(1-p)\bar{u}_2}{d} + \frac{\varepsilon}{2} = \bar{v}_2, \quad t \geq T_6. \tag{4.20}$$

From (4.13), (4.16) and (4.20), we obtain

$$\bar{v}_2 < \bar{v}_1. \tag{4.21}$$

From (4.20) and the first equation of system (1.3), we obtain

$$\dot{x}_m \geq \alpha^{-\gamma\tau} x_m(t-\tau) - \beta x_m^2(t) - \frac{x_m(t)\bar{v}_2}{x_m(t) + \frac{\bar{v}_2}{1-p}}, \quad t > T_6 + \tau.$$

By the similar arguments as above and Lemma 4.1, for any sufficiently small $\varepsilon > 0$, there exists a $T_7 > T_6 + \tau > 0$ such that

$$x_m(t) \geq \frac{\alpha e^{-\gamma\tau}}{2\beta} - \frac{\overline{v}_2}{2(1-p)} + \frac{\sqrt{\Big(\alpha^{-\gamma\tau} - \dfrac{\beta\overline{v}_2}{1-p}\Big)^2 + 4\beta\Big(\dfrac{\alpha^{-\gamma\tau}}{1-p} - 1\Big)\overline{v}_2}}{2\beta} - \frac{\varepsilon}{2} = \underline{u}_2. \tag{4.22}$$

According to (4.15), (4.21) and (4.22), we obtain

$$\underline{u}_2 > \underline{u}_1. \tag{4.23}$$

From the above inequality and the second equation of system (1.3), we obtain

$$\dot{y}(t) \geq \frac{k(1-p)\underline{u}_2 y(t) - dy^2(t)}{(1-p)\underline{u}_2 + \overline{v}_2}, \quad t \geq T_7.$$

By the comparison theorem and Lemma 2.3, there exists a $T_8 > T_7$ such that

$$y(t) \geq \frac{(k-d)(1-p)\underline{u}_2}{d} - \frac{\varepsilon}{2} = \underline{v}_2, \quad t \geq T_8. \tag{4.24}$$

From (4.16), (4.23) and (4.24), we obtain

$$\underline{v}_2 > \underline{v}_1. \tag{4.25}$$

Repeating the above steps can obtain four sequences $(\overline{u}_n)_{n=1}^{\infty}$, $(\underline{u}_n)_{n=1}^{\infty}$, $(\underline{v}_n)_{n=1}^{\infty}$, $(\overline{v}_n)_{n=1}^{\infty}$, and $T_{4n} > 0$. For $t \geq T_{4n}$, we can obtain that

$$0 < \underline{u}_1 < \underline{u}_2 < \cdots < \underline{u}_n < x_m(t) < \overline{u}_n < \cdots < \overline{u}_2 < \overline{u}_1,$$
$$0 < \underline{v}_1 < \underline{v}_2 < \cdots < \underline{v}_n < y(t) < \overline{v}_n < \cdots < \overline{v}_2 < \overline{v}_1.$$

Hence, the limits of $(\overline{u}_n)_{n=1}^{\infty}$, $(\underline{u}_n)_{n=1}^{\infty}$, $(\underline{v}_n)_{n=1}^{\infty}$, $(\overline{v}_n)_{n=1}^{\infty}$ exist. Set

$$\overline{u} = \lim_{n\to\infty} \overline{u}_n, \quad \overline{v} = \lim_{n\to\infty} \overline{v}_n, \quad \underline{u} = \lim_{n\to\infty} \underline{u}_n, \quad \underline{v} = \lim_{n\to\infty} \underline{v}_n.$$

Then $\overline{u} \geq \underline{u}$, $\overline{v} \geq \underline{v}$.

According to the relationship between u_n and v_n, we can know

$$\overline{v}_n - \underline{v}_n = \frac{(k-d)(1-p)(\overline{u}_n - \underline{u}_n)}{2\beta d}. \tag{4.26}$$

From the definitions of $\overline{u}_n$ and $\underline{u}_n$, we obtain

$$\begin{aligned}\overline{u}_n - \underline{u}_n = {} & \frac{\overline{v}_n - \underline{v}_{n-1}}{2(1-p)} + \frac{\sqrt{\Big(\alpha^{-\gamma\tau} - \dfrac{\beta\underline{v}_{n-1}}{1-p}\Big)^2 + 4\beta\Big(\dfrac{\alpha e^{-\gamma\tau}}{1-p} - 1\Big)\underline{v}_{n-1}}}{2\beta} \\ & - \frac{\sqrt{\Big(\alpha^{-\gamma\tau} - \dfrac{\beta\overline{v}_n}{1-p}\Big)^2 + 4\beta\Big(\dfrac{\alpha e^{-\gamma\tau}}{1-p} - 1\Big)\overline{v}_n}}{2\beta} + \frac{2\varepsilon}{n}\end{aligned}$$

$$
\begin{aligned}
&< \frac{\Big(2\alpha^{-\gamma\tau} - \dfrac{\beta(\overline{v}_n + \underline{v}_{n-1})}{1-p}\Big)\dfrac{\beta}{1-p}(\overline{v}_n - \underline{v}_{n-1})}{2\beta\Big(2\alpha^{-\gamma\tau} - \dfrac{\beta(\overline{v}_n + \underline{v}_{n-1})}{1-p}\Big)} \\
&\quad - \frac{4\beta\Big(\dfrac{\alpha e^{-\gamma\tau}}{1-p} - 1\Big)(\overline{v}_n - \underline{v}_{n-1})}{2\beta\Big(2\alpha^{-\gamma\tau} + \dfrac{\beta(\overline{v}_n + \underline{v}_{n-1})}{1-p}\Big)} + \frac{\overline{v}_n - \underline{v}_{n-1}}{2(1-p)} + \frac{2\varepsilon}{n} \\
&< \frac{\overline{v}_n - \underline{v}_{n-1}}{2(1-p)} + \frac{\overline{v}_n - \underline{v}_{n-1}}{2(1-p)} - \frac{2\Big(\dfrac{\alpha^{-\gamma\tau}}{1-p} - 1\Big)(\overline{v}_n - \underline{v}_{n-1})}{2\alpha e^{-\gamma\tau} + \dfrac{\beta(\overline{v}_n + \underline{v}_{n-1})}{1-p}} + \frac{2\varepsilon}{n} \\
&< \frac{\overline{v}_n - \underline{v}_{n-1}}{1-p} + \frac{2\varepsilon}{n}. \qquad (4.27)
\end{aligned}
$$

Hence,

$$
\begin{aligned}
\overline{v}_n - \underline{v}_n &= \frac{(k-d)(1-p)(\overline{u}_n - \underline{u}_n)}{d} + \frac{2\varepsilon}{n} \\
&< \frac{(k-d)(1-p)}{d}\Big(\frac{\overline{v}_n - \underline{v}_{n-1}}{1-p} + \frac{2\varepsilon}{n}\Big) + \frac{2\varepsilon}{n} \\
&< \frac{k-d}{d}(\overline{v}_n - \underline{v}_{n-1}) + \frac{2\varepsilon(k-d)(1-p)}{nd} + \frac{2\varepsilon}{n}. \qquad (4.28)
\end{aligned}
$$

Taking the limit at the same time on both sides of the inequality, we can obtain

$$\overline{v} - \underline{v} \leq \frac{k-d}{d}(\overline{v} - \underline{v}), \qquad (4.29)$$

$$\Big(1 - \frac{k-d}{d}\Big)\overline{v} - \underline{v} \leq 0. \qquad (4.30)$$

According to the assumed conditions we have $\overline{v} = \underline{v}$, which implies $\overline{u} = \underline{u}$. Hence, we obtain when $k < 2d$ and $\alpha e^{-\gamma\tau} > 1 - p$, the equilibrium point B is globally attractive.

5 Numerical Simulations

The following four examples show the feasibility of main results.

Example 5.1 Consider the following system

$$
\begin{aligned}
\dot{x}_m(t) &= 3e^{-2.5}x_m(t-10) - x_m^2(t) - \frac{0.2x_m(t)y(t)}{0.2x_m(t) + y(t)}, \\
\dot{y}(t) &= 0.4\frac{0.2x_m(t)y(t)}{0.2x_m(t) + y(t)} - 2y(t),
\end{aligned} \qquad (5.1)
$$

where $\alpha = 3, \gamma = 0.25, \beta = 1, k = 0.4, d = 2, p = 0.8, \tau = 10$. By direct calculation, $k < d$, $\alpha e^{-\gamma\tau} = 3e^{-2.5} \approx 0.246 > 1 - p = 0.2$. It follows from Theorem 4.1 that the boundary equilibrium A is globally asymptotically stable.

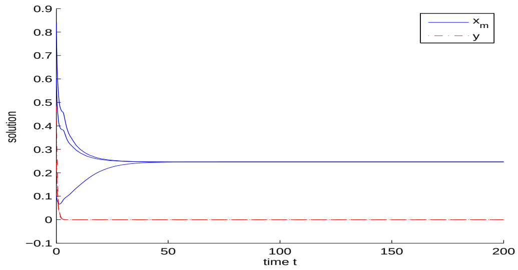

Figure 1: Dynamic behaviors of system (5.1) with the initial conditions $(x_m(\theta), y(\theta))$=(0.1; 0.5), (0.6; 0.25) and (0.3; 0.6) for $-10 \le \theta < 0$.

Example 5.2 Consider the following system

$$\begin{aligned} \dot{x}_m(t) &= 3e^{-1}x_m(t-5) - x_m^2(t) - \frac{(1-p)x_m(t)y(t)}{(1-p)x_m(t)+y(t)}, \\ \dot{y}(t) &= 0.8\frac{(1-p)x_m(t)y(t)}{(1-p)x_m(t)+y(t)} - 0.5y(t), \end{aligned} \tag{5.2}$$

where $\alpha = 3, \gamma = 0.2, \beta = 1, k = 0.8, d = 0.5, \tau = 5$. Fixed $p = 0.8$ by calculation, $3e^{-1} \approx 1.103 > 1 - p = 0.2$ and $d < k < 2d$. It follows from Theorem 4.2 that the positive equilibrium $(1.1096, 0.131)$ of system (5.2) is globally attractive (see Figure 2). Clearly, x_m^* and y^* are the functions of $p \in (0, 1)$. Since $0.8 = 2(k - d) > k\alpha e^{-\gamma\tau} = 0.441$, it follows from the analysis of Section 3 that increasing the number of prey refuge can increase the prey densities and decrease the predator densities. Figure 3 supports our results.

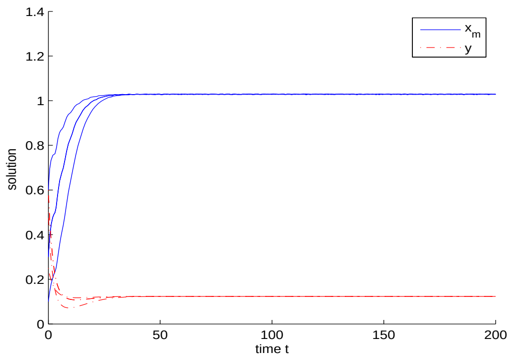

Figure 2: Dynamic behaviors of system (5.2) with the initial conditions $(0.1, 0.5), (0.6; 0.25)$ and $(0.3, 0.6)$ for $-5 \le \theta < 0$.

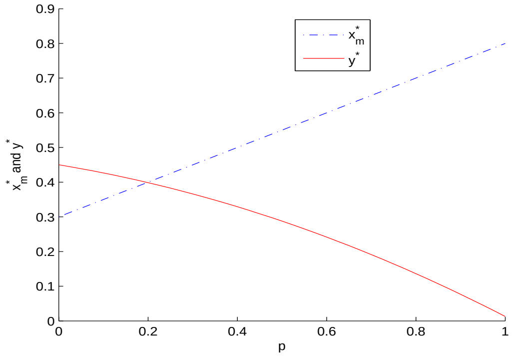

Figure 3: Numeric simulations of $x_m^*(p), y^*(p)$, where $p \in (0,1)$.

Example 5.3 Consider the following system

$$\begin{aligned}\dot{x}_m(t) &= 1.5\mathrm{e}^{-1}x_m(t-5) - x_m^2(t) - \frac{(1-p)x_m(t)y(t)}{(1-p)x_m(t)+y(t)},\\ \dot{y}(t) &= 0.9\frac{(1-p)x_m(t)y(t)}{(1-p)x_m(t)+y(t)} - 0.5y(t),\end{aligned} \tag{5.3}$$

where $\alpha = 1.5$, $\gamma = 0.2$, $\beta = 1$, $k = 0.9$, $d = 0.5$, $\tau = 5$, $p \in [0,1]$. In this case, $0.8 = 2(k-d) \leq k\alpha\mathrm{e}^{-\gamma\tau} = 0.441$. From the analysis in Section 3, we obtain when $p < p^* = (k\alpha\mathrm{e}^{-\gamma\tau} + 2d - 2k)/[2(d-k)] = 0.449$, increasing the number of prey refuge can increase the prey densities and the predator densities. However, when $p > p^* = (k\alpha\mathrm{e}^{-\gamma\tau} + 2d - 2k)/[2(d-k)] = 0.449$, increasing the number of prey refuge can decrease the predator densities. Figure 4 supports our results.

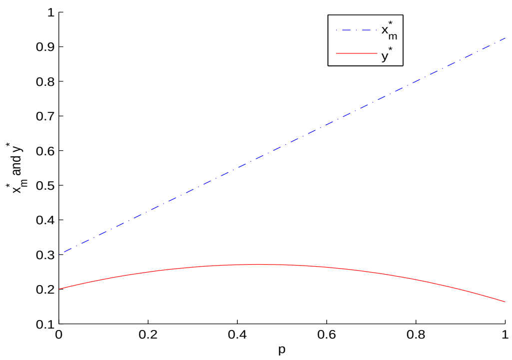

Figure 4: Numeric simulations of $x_m^*(p), y^*(p)$, where $p \in (0,1)$.

Example 5.4 Consider the following system

$$\begin{aligned}\dot{x}_m(t) &= 0.1\mathrm{e}^{-1}x_m(t-5) - x_m^2(t) - \frac{0.2x_m(t)y(t)}{0.2x_m(t)+y(t)},\\ \dot{y}(t) &= 0.8\frac{0.2x_m(t)y(t)}{0.2x_m(t)+y(t)} - 0.5y(t),\end{aligned} \tag{5.4}$$

where $\alpha = 0.1$, $\gamma = 0.2$, $\beta = 1$, $k = 0.8$, $d = 0.5$, $p = 0.8$, $\tau = 5$. In this case, $0.1e^{-1} \approx 0.036 < 1 - p = 0.2$. The numeric simulation (see Figure 5) shows that the predator and prey species all will be possible driven into extinction.

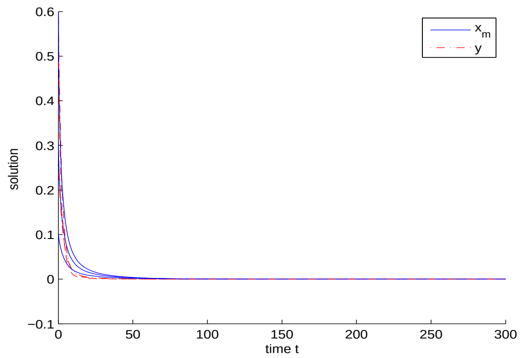

Figure 5: Dynamic behaviors of system (5.4) with the initial conditions $(0.1, 0.5), (0.6; 0.25)$ and $(0.3, 0.6)$ for $-5 \leq \theta < 0$.

6 Conclusion

In this paper, we study the dynamic behaviors of the predator-prey system with a constant proportion of prey refuge and stage-structure for prey species. We study the permanence, the local stability and global stability of the system. Our results show that the prey refuge plays an important role in determining the persistence and the stability property of the system. Theorems 2.2, 3.2 and 4.2 show that if prey refuge is large enough, then system (1.3) is permanent and the unique positive equilibrium of the system is locally and globally stable, which means that the two species of system could be coexistence in a stable state.

Furthermore, compared with the results of Devi [11], we found that the constant proportion of prey refuge could lead to more complicate dynamic behaviors than the fixed constant of prey refuges. The equilibrium value of mature prey populations increases with p, as far as predator species is concerned. If $2(k - d) \leq k\alpha e^{-\gamma\tau}$, then increasing the number of prey refuge can decrease the predator densities. If $2(k - d) > k\alpha e^{-\gamma\tau}$, there is a threshold p^*, when the prey refuge smaller than this threshold, increasing the amount of prey refuge can increase the predator species, but when the prey refuge is larger than the threshold, increasing the amount of prey refuge can decrease the predator densities. Numeric simulations also support our findings.

At the end of the paper, we would like to mention that Theorems 2.2 and 4.2 show that $\alpha e^{-\gamma\tau} > 1 - p$ plays an important role on the persistent and stability property of the system. Noting that τ represents the period of the prey species from immature to mature, shorten the period between the mature and immature prey species will improve the chance of the coexistence of the two species. However, what would happen under the assumption $\alpha e^{-\gamma\tau} < 1 - p$? Example 5.4 shows that in this case, both predator and prey species will be driven to extinction. However, at present we have difficulty in giving its strictly proof, we leave this for future investigation.

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