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PERMANENCE OF AN IMPULSIVE PREDATOR-PREY SYSTEM WITH MUTUAL INTERFERENCE AND CROWLEY-MARTIN RESPONSE FUNCTION*†

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Abstract

In this paper, we investigated an impulsive predator-prey model with mutual interference and Crowley-Martin response function. By the comparison theorem and the analysis technique of [12,14], sufficient conditions for the permanence of the impulsive model are obtained, which generalizes one of main results of [4].

Keywords predator-prey; mutual interference; Crowley-Martin; impulsive; permanence

2010 Mathematics Subject Classification 34C25; 92D25; 34D20; 34D40

1 Introduction

In 1989, Crowley and Martin [1] proposed a functional response which can accommodate interference among predators, for high predator density and handling or searching of prey by predator individual. The per capita feeding rate in this formulation can be written as follows:

$$\varphi(x_1, x_2) = \frac{cx_1}{1 + ex_1 + fx_2 + efx_1x_2},$$

where c, e, f can be interpreted as the effects of capture rate, handing time, the magnitude of interference among predators respectively, on the feeding rate. All

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the coefficients c, e, f are positive constants. We can easily obtain that $\varphi(x_1, x_2)$ is positively correlated with x_1 and varies inversely with respect to x_2.

The following autonomous predator-prey system with Crowley-Martin response functional was proposed and studied:

$$\begin{aligned} \dot{x}_1(t) &= x_1(t)\Big(a_1 - b_1x_1(t) - \frac{c_1x_2(t)}{d + ex_1(t) + fx_2(t) + gx_1(t)x_2(t)}\Big), \\ \dot{x}_2(t) &= x_2(t)\Big(-a_2 + \frac{c_2x_1(t)}{d + ex_1(t) + fx_2(t) + gx_1(t)x_2(t)}\Big), \end{aligned} \tag{1.1}$$

in [2], where the local stability of the equilibria, global asymptotic stability of the positive equilibrium and permanence were carried out in system (1.1). [3] discussed a predator-prey model with Crowley-Martin functional response and density dependent predator:

$$\begin{aligned} \dot{x}_1(t) &= x_1(t)\Big(a_1 - b_1x_1(t) - \frac{c_1x_2}{d + ex_1(t) + fx_2(t) + gx_1(t)x_2(t)}\Big), \\ \dot{x}_2(t) &= x_2(t)\Big(-a_2 - b_2x_2(t) + \frac{c_2x_1(t)}{d + ex_1(t) + fx_2(t) + gx_1(t)x_2(t)}\Big). \end{aligned} \tag{1.2}$$

They considered the permanence, non-permanence, local asymptotic stability behavior of various equilibrium points and global asymptotic stability of positive equilibrium to understand the dynamics of both delayed and non-delayed model systems.

On account of the fluctuation in many biological or environmental parameters as time goes on. And the prey has the tendency to leave each other when they meet, which interferes with predators capture effects. Recently, Tripathi [4] considered and explored the almost periodic solution and global attractivity of the following two dimensional non-autonomous, density dependent predator-prey model with mutual interference and Crowley-Martin response function:

$$\begin{aligned} \dot{x}_1(t) &= x_1(t)\Big(a_1(t) - b_1(t)x_1(t) - \frac{c_1(t)x_2^{\beta}(t)}{d(t) + e(t)x_1(t) + f(t)x_2(t) + g(t)x_1(t)x_2(t)}\Big), \\ \dot{x}_2(t) &= x_2(t)\Big(-a_2(t) - b_2(t)x_2(t) + \frac{c_2(t)x_1(t)x_2^{\beta-1}(t)}{d(t) + e(t)x_1(t) + f(t)x_2(t) + g(t)x_1(t)x_2(t)}\Big), \end{aligned} \tag{1.3}$$

However, the ecosystem is often deeply perturbed by nature and human exploit activities such as drought, fire, flooding deforestation, hunting, harvesting, breeding and so forth. For the sake of accurately describing the real-world phenomena, impulsive differential equations may be a better candidate than ordinary differential equations or difference equations. Motivated by these facts, we propose the following impulsive predator-prey model with mutual interference and Crowley-Martin response function:

$$\begin{aligned}
\dot{x}_1(t) &= x_1(t)\Big(a_1(t) - b_1(t)x_1(t) - \frac{c_1(t)x_2^{\beta}(t)}{d(t)+e(t)x_1(t)+f(t)x_2(t)+g(t)x_1(t)x_2(t)}\Big),\\
\dot{x}_2(t) &= x_2(t)\Big(-a_2(t) - b_2(t)x_2(t) + \frac{c_2(t)x_1(t)x_2^{\beta-1}(t)}{d(t)+e(t)x_1(t)+f(t)x_2(t)+g(t)x_1(t)x_2(t)}\Big),\\
x_1(t_k^+) &= h_{1k}x_1(t_k), \quad x_2(t_k^+) = h_{2k}x_2(t_k), \quad k=1,2,\cdots.
\end{aligned} \tag{1.4}$$

The objective of this paper is to analyze the permanence property of system (1.4) by utilizing the comparison theorem and developing the analysis technique of [12, 14].

This paper is organized as follows: In Section 2, we introduce the assumptions and preliminary lemmas. In Section 3, we give some conditions for the permanence of system (1.4). In Section 4, we present numerical simulations to illustrate our main results.

2 Preliminaries

In this section, we shall state two lemmas and some assumptions which will be useful for proving our main results. Functions $a_i(t), b_i(t), c_i(t)$ $(i=1,2)$ and $d(t), e(t), f(t), g(t)$ are continuous and bounded above and below by positive constants; for a continuous function $y(t)$, let y_L and y_M denote $\inf\limits_{t\in(-\infty,+\infty)} y(t)$ and $\sup\limits_{t\in(-\infty,+\infty)} y(t)$, respectively. For impulsive perturbations h_{ik}, let h_{ikL} and h_{ikM} denote $\inf\limits_{k\in Z} h_{ik}$ and $\sup\limits_{k\in Z} h_{ik}$, respectively. For the k-th impulse points t_k, denote $0 < \inf\limits_{k\in Z} t_k^1 = \inf\limits_{k\in Z}(t_{k+1}-t_k) = \theta \le \sup\limits_{k\in Z} t_k^1 = \eta$.

Lemma 1[2] *Assume that $a\theta + \ln h_{kL} > 0$, then for any positive solution $x(t)$ of the following autonomous Logistic system*

$$\begin{aligned}
\dot{x}(t) &= x(t)(a - bx(t)),\\
x(t_k^+) &= h_k x(t_k), \quad k=1,2,\cdots,
\end{aligned}$$

where a and b are positive bounded constants,

$$m_0 \le \liminf_{t\to+\infty} x(t) \le \limsup_{t\to+\infty} x(t) \le M_0,$$

with

$$M_0 = \min\left\{\frac{a\eta + \ln h_{kM}}{b\eta h_{kM}}, \frac{(a\theta + \ln h_{kM})h_{kM}}{b\theta}\right\},$$

$$m_0 = \min\left\{\frac{a\eta + \ln h_{kL}}{b\eta h_{kL}}, \frac{(a\theta + \ln h_{kL})h_{kL}}{b\theta}\right\}.$$

Consider the following impulsive logistic system

$$\begin{aligned} \dot{x}(t) &= x(t)(a - bx^{\beta}(t)), \\ x(t_k^+) &= h_k x(t_k), \quad k = 1, 2, \cdots, \end{aligned} \tag{2.1}$$

where a and b are positive bounded constants.

Lemma 2 *Suppose that:*

$$\beta a\theta + \ln h_{kL}^{\beta} > 0. \tag{2.2}$$

Then for any positive solution $x(t)$ of system (2.1)*,*

$$m \le \liminf_{t\to+\infty} x(t) \le \limsup_{t\to+\infty} x(t) \le M,$$

where

$$M = \min\left\{\left(\frac{a\eta + \ln h_{kM}}{b\eta h_{kM}^{\beta}}\right)^{\frac{1}{\beta}}, \left(\frac{(a\theta + \ln h_{kM})h_{kM}^{\beta}}{b\theta}\right)^{\frac{1}{\beta}}\right\},$$

$$m = \min\left\{\left(\frac{a\eta + \ln h_{kL}}{b\eta h_{kL}^{\beta}}\right)^{\frac{1}{\beta}}, \left(\frac{(a\theta + \ln h_{kL})h_{kL}^{\beta}}{b\theta}\right)^{\frac{1}{\beta}}\right\}.$$

Proof Let $y(t) = x^{\beta}(t)$, then system (2.1) is transformed into

$$\begin{aligned} \dot{y}(t) &= y(t)(\beta a - \beta b y(t)), \\ y(t_k^+) &= h_k^{\beta} y(t_k), \quad k = 1, 2, \cdots. \end{aligned}$$

Note that condition (2.2) implies $\beta a\theta + \ln h_{kL}^{\beta} > 0$.

According to Lemma 1, we obtain

$$\liminf_{t\to+\infty} y(t) \ge \min\left\{\frac{a\eta + \ln h_{kL}}{b\eta h_{kL}^{\beta}}, \frac{(a\theta + \ln h_{kL})h_{kL}^{\beta}}{b\theta}\right\},$$

and

$$\limsup_{t\to+\infty} y(t) \le \min\left\{\frac{a\eta + \ln h_{kM}}{b\eta h_{kM}^{\beta}}, \frac{(a\theta + \ln h_{kM})h_{kM}^{\beta}}{b\theta}\right\}.$$

Hence

$$\liminf_{t\to+\infty} x(t) \ge \min\left\{\left(\frac{a\eta + \ln h_{kL}}{b\eta h_{kL}^{\beta}}\right)^{\frac{1}{\beta}}, \left(\frac{(a\theta + \ln h_{kL})h_{kL}^{\beta}}{b\theta}\right)^{\frac{1}{\beta}}\right\} \overset{\text{def}}{=} m,$$

and

$$\limsup_{t\to+\infty} x(t) \le \min\left\{\left(\frac{a\eta + \ln h_{kM}}{b\eta h_{kM}^{\beta}}\right)^{\frac{1}{\beta}}, \left(\frac{(a\theta + \ln h_{kM})h_{kM}^{\beta}}{b\theta}\right)^{\frac{1}{\beta}}\right\} \overset{\text{def}}{=} M.$$

3 Permanence

Theorem 1 *Assume that*

$$\Big(a_{1L} - \frac{c_{1M}M_2^{\beta}}{d_L}\Big)\theta + \ln h_{1kL} > 0, \tag{H$_1$}$$

$$(a_{2M} - \xi)\theta < \ln h_{2kL} < a_{2L}\theta, \tag{H$_2$}$$

then for any positive solution $Y(t) = (x_1(t), x_2(t))^{\mathrm{T}}$ *of system* (1.4),

$$m_i \le \liminf_{t\to+\infty} x_i(t) \le \limsup_{t\to+\infty} x_i(t) \le M_i, \quad i = 1, 2,$$

where

$$\begin{aligned}
M_1 &= \min\Big\{\frac{a_{1M}\eta + \ln h_{1kM}}{b_{1L}\eta h_{1kM}}, \frac{(a_{1M}\theta + \ln h_{1kM})h_{1kM}}{b_{1L}\theta}\Big\},\\
M_2 &= \min\Big\{\Big(\frac{a_{2L}\eta - \ln h_{2kL}}{\lambda\eta h_{2kL}^{\beta-1}}\Big)^{\frac{1}{\beta-1}}, \Big(\frac{(a_{2L}\theta - \ln h_{2kL})h_{2kL}^{\beta-1}}{\lambda\theta}\Big)^{\frac{1}{\beta-1}}\Big\},\\
m_1 &= \min\Big\{\frac{(a_{1L} - \chi)\eta + \ln h_{1kL}}{b_{1M}\eta h_{1kL}}, \frac{((a_{1L} - \chi)\theta + \ln h_{1kL})h_{1kL}}{b_{1M}\theta}\Big\},\\
m_2 &= \min\Big\{\frac{(-a_{2M} + \xi)\eta + \ln h_{2kL}}{b_{2M}\eta h_{2kL}}, \frac{((-a_{2M} + \xi)\theta + \ln h_{2kL})h_{2kL}}{b_{2M}\theta}\Big\},
\end{aligned}$$

with

$$\lambda = \frac{c_{2M}M_1}{d_L + e_L M_1}, \quad \chi = \frac{c_{1M}M_2^{\beta}}{d_L}, \quad \xi = \frac{c_{2L}m_1 M_2^{\beta-1}}{d_M + e_M m_1 + f_M M_2 + g_M m_1 M_2}.$$

Proof By (H$_2$), it is obvious that

$$a_{2L}\theta - \ln h_{2kL} > 0, \tag{3.1}$$

$$(-a_{2M} + \xi)\theta + \ln h_{2kL} > 0. \tag{3.2}$$

From the first equation of system (1.4), we have

$$\dot{x}_1(t) \le x_1(t)(a_{1M} - b_{1L}x_1(t)),$$

for all $t \ne t_k$.

Applying the comparison theorem, we can obtain $x_1(t) \le \omega(t)$, where $\omega(t)$ is the maximal solution of the scalar impulsive differential equation

$$\begin{aligned}
&\dot{\omega}(t) = \omega(t)(a_{1M} - b_{1L}\omega(t)),\\
&\omega(t_k^+) = h_{1k}\omega(t_k), \quad k = 1, 2, \cdots.
\end{aligned}$$

From (H_1), it is easy to see $a_{1M}\theta + \ln h_{1kL} > 0$. Using Lemma 1, we obtain

$$\limsup_{t\to+\infty} x_1(t) \le \min\left\{\frac{a_{1M}\eta + \ln h_{1kM}}{b_{1L}\eta h_{1kM}}, \frac{(a_{1M}\theta + \ln h_{1kM})h_{1kM}}{b_{1L}\theta}\right\} \stackrel{\text{def}}{=} M_1.$$

Therefore, for arbitrary $\varepsilon_1 > 0$ there exists a positive real number T_1 such that

$$x_1(t) < M_1 + \varepsilon_1, \tag{3.3}$$

for all $t > T_1$.

Consider the transformation $z(t) = \frac{1}{x_2(t)}$, and from the second equations of system (1.4) and (3.3), it is easy to obtain

$$\begin{aligned}\dot{z}(t) &= z(t)\left(a_2(t) + b_2(t)\frac{1}{z(t)} - \frac{c_2(t)x_1(t)z(t)}{d(t)z(t) + e(t)x_1(t)z(t) + f(t) + g(t)x_1(t)} z^{1-\beta}(t)\right)\\ &\ge z(t)\left(a_{2L} - \frac{c_{2M}(M_1+\varepsilon_1)}{d_L + e_L(M_1+\varepsilon_1)} z^{1-\beta}(t)\right),\end{aligned}$$

for all $t > T_1$ and $t \ne t_k$.

Setting $\varepsilon_1 \to 0$, in the above expression, we have

$$\begin{aligned}\dot{z}(t) &\ge z(t)\left(a_{2L} - \frac{c_{2M}M_1}{d_L + e_L M_1} z^{1-\beta}(t)\right),\\ z(t_k^+) &= \frac{1}{h_{2k}} z(t_k), \quad k = 1, 2, \cdots.\end{aligned}$$

According to the comparison theorem, we can obtain $z(t) \ge \mu(t)$, where $\mu(t)$ is any positive solution of the following system

$$\begin{aligned}\dot{\mu}(t) &= \mu(t)\left(a_{2L} - \frac{c_{2M}M_1}{d_L + e_L M_1} \mu^{1-\beta}(t)\right),\\ \mu(t_k^+) &= \frac{1}{h_{2k}} \mu(t_k), \quad k = 1, 2, \cdots.\end{aligned}$$

Applying Lemma 2 and inequality (3.1), we have

$$\liminf_{t\to+\infty} z(t) \ge \liminf_{t\to+\infty} \mu(t) \ge \min\left\{\left(\frac{a_{2L}\eta - \ln h_{2kL}}{\lambda\eta h_{2kL}^{\beta-1}}\right)^{\frac{1}{1-\beta}}, \left(\frac{(a_{2L}\theta - \ln h_{2kL})h_{2kL}^{\beta-1}}{\lambda\theta}\right)^{\frac{1}{1-\beta}}\right\},$$

here $\lambda = \frac{c_{2M}M_1}{d_L + e_L M_1}$, which further gives that

$$\limsup_{t\to+\infty} x_2(t) \le \min\left\{\left(\frac{a_{2L}\eta - \ln h_{2kL}}{\lambda\eta h_{2kL}^{\beta-1}}\right)^{\frac{1}{\beta-1}}, \left(\frac{(a_{2L}\theta - \ln h_{2kL})h_{2kL}^{\beta-1}}{\lambda\theta}\right)^{\frac{1}{\beta-1}}\right\} \stackrel{\text{def}}{=} M_2. \tag{3.4}$$

In view of (3.4), for any $\varepsilon_2 > 0$ small enough, there exists a $T_2 > T_1$ such that

$$x_2(t) < M_2 + \varepsilon_2, \tag{3.5}$$

for all $t > T_2$.

Furthermore, from the first equation of system (1.4), we have

$$\begin{aligned}\dot{x}_1(t) &\geq x_1(t)\left(a_{1L} + b_{1M}x_1(t) - \frac{c_{1M}(M_2+\varepsilon_2)^\beta}{d_{1L}}\right)\\ &= x_1(t)\left[\left(a_{1L} - \frac{c_{1M}(M_2+\varepsilon_2)^\beta}{d_L}\right) + b_{1M}x_1(t)\right],\end{aligned}$$

for all $t > T_2$ and $t \neq t_k$. Setting $\varepsilon_2 \to 0$ in the above expression, we have

$$\begin{aligned}&\dot{x}_1(t) \geq x_1(t)\big((a_{1L} - \chi) - b_{1M}x_1(t)\big),\\ &x_1(t_k^+) = h_{1k}x_1(t_k), \quad k = 1, 2, \cdots,\end{aligned}$$

where $\chi = \frac{c_{1M}M_2^\beta}{d_L}$. From (H_1), according to the comparison theorem and Lemma 1, we obtain

$$\liminf_{t\to+\infty} x_1(t) \geq \min\left\{\frac{(a_{1L}-\chi)\eta + \ln h_{1kL}}{b_{1M}\eta h_{1kL}}, \frac{((a_{1L}-\chi)\theta + \ln h_{1kL})h_{1kL}}{b_{1M}\theta}\right\} \stackrel{\text{def}}{=} m_1. \tag{3.6}$$

Hence, from (3.6), for any $\varepsilon_3 > 0$ small enough, there exists a $T_3 > T_2$ such that

$$x_1(t) > m_1 - \varepsilon_3, \tag{3.7}$$

for all $t > T_3$. Using (3.5) and (3.7) in the second equation of system (1.4) one obtains that

$$\dot{x}_2(t) \geq x_2(t)\left[\left(-a_{2M} + \frac{c_{2L}(m_1-\varepsilon_3)(M_2+\varepsilon_2)^{\beta-1}}{d_M + e_M(m_1-\varepsilon_3) + f_M(M_2+\varepsilon_2) + g_M(m_1-\varepsilon_3)(M_2+\varepsilon_2)}\right) - b_{2M}x_2(t)\right],$$

for all $t > T_3$ and $t \neq t_k$.

From the second and forth equation of system (1.4), and by setting $\varepsilon_2, \varepsilon_3 \to 0$ in the above expression, we have

$$\begin{aligned}&\dot{x}_2(t) \geq x_2(t)\big((-a_{2M} + \xi) - b_{2M}x_2(t)\big),\\ &x_2(t_k^+) = h_{2k}x_2(t_k), \quad k = 1, 2, \cdots,\end{aligned}$$

where $\xi = \frac{c_{2L}m_1M_2^{\beta-1}}{d_M + e_M m_1 + f_M M_2 + g_M m_1 M_2}$. According to inequality (3.2) and applying Lemma 1, the above inequality implies that

$$\liminf_{t\to+\infty} x_2(t) \geq \min\left\{\frac{(-a_{2M}+\xi)\eta + \ln h_{2kL}}{b_{2M}\eta h_{2kL}}, \frac{((-a_{2M}+\xi)\theta + \ln h_{2kL})h_{2kL}}{b_{2M}\theta}\right\} \stackrel{\text{def}}{=} m_2.$$

The proof is completed.

Theorem 2 *When $h_{1k} = h_{2k} = 1$ for all $t \geq 0$ in system* (1.4)*, system* (1.4) *is reduced to* (1.3)*. Suppose that the following inequalities:*

$$a_{1L} > \frac{c_{1M} M_2^{\beta}}{d_L}, \quad \frac{c_{2L} m_1 M_2^{\beta-1}}{d_M + e_M m_1 + f_M M_2 + g_M m_1 M_2} > a_{2M},$$

then system (1.3) *is permanent.*

That is one of main results in [4].

4 Numeric Simulations

In the following, we give a numerical example to illustrate the feasibilities of our analytical results.

Example 4.1 Consider system (1.4) with the following coefficients:

$$\begin{aligned}
&h_{1k} = 2.25, \quad h_{2k} = 1.05, \quad \beta = 0.2, \quad a_1(t) = 2.25, \quad b_1(t) = 2.2 + 0.01\cos(\sqrt{3}t),\\
&c_1(t) = 0.005, \quad a_2(t) = 0.1667 + 0.0001\sin(\sqrt{4}t), \quad b_2(t) = 3.8 + 0.01\sin(t),\\
&c_2(t) = 1.01, \quad d(t) = 0.54 - 0.001\cos(\sqrt{4}t), \quad e(t) = 3.38 + 0.001\sin(t),\\
&f(t) = 0.005 + 0.001\cos(\sqrt{2}t), \quad g(t) = 0.01 + 0.001\sin(t).
\end{aligned}$$

It is obvious that

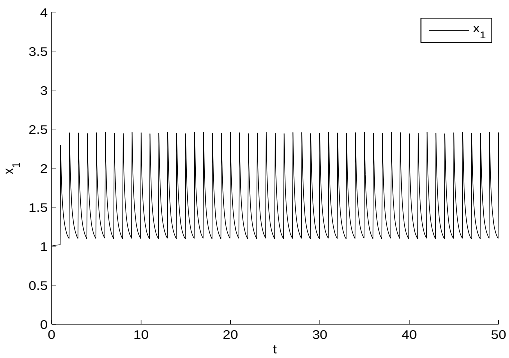
$$\begin{aligned}
M_1 &= \min\left\{\frac{a_{1M}\eta + \ln h_{1kM}}{b_{1L}\eta h_{1kM}}, \frac{(a_{1M}\theta + \ln h_{1kM})h_{1kM}}{b_{1L}\theta}\right\} = \frac{2.25 + \ln 2.25}{2.19 \times 2.25} \approx 0.6212,\\
\lambda &= \frac{c_{2M} M_1}{d_L + e_L M_1} = \frac{1.01 \times 0.6212}{0.539 + 3.379 \times 0.6212} \approx 0.2378,\\
M_2 &= \min\left\{\left(\frac{a_{2L}\eta - \ln h_{2kL}}{\lambda\eta h_{2kL}^{\beta-1}}\right)^{\frac{1}{\beta-1}}, \left(\frac{(a_{2L}\theta - \ln h_{2kL})h_{2kL}^{\beta-1}}{\lambda\theta}\right)^{\frac{1}{\beta-1}}\right\}\\
&= \left(\frac{0.1666 - \ln 1.05}{0.2378 \times 1.05^{-0.8}}\right)^{-\frac{1}{0.8}} \approx 2.2914,\\
\chi &= \frac{c_{1M} M_2^{\beta}}{d_L} = \frac{0.005 \times 2.2914^{0.2}}{0.539} \approx 0.0109,\\
m_1 &= \min\left\{\frac{(a_{1L} - \chi)\eta + \ln h_{1kL}}{b_{1M}\eta h_{1kL}}, \frac{((a_{1L} - \chi)\theta + \ln h_{1kL})h_{1kL}}{b_{1M}\theta}\right\}\\
&= \frac{2.25 - 0.0109 + \ln 2.25}{2.21 \times 2.25} \approx 0.6134,\\
\xi &= \frac{c_{2L} m_1 M_2^{\beta-1}}{d_M + e_M m_1 + f_M M_2 + g_M m_1 M_2}\\
&= \frac{1.01 \times 0.6134 \times 2.2914^{-0.8}}{0.541 + 3.381 \times 0.6134 + 0.006 \times 2.2914 + 0.011 \times 0.6134 \times 2.2914} \approx 0.1207,
\end{aligned}$$

$$\begin{aligned} m_2 &= \min\left\{\frac{(-a_{2M}+\xi)\eta+\ln h_{2kL}}{b_{2M}\eta h_{2kL}}, \frac{((-a_{2M}+\xi)\theta+\ln h_{2kL})h_{2kL}}{b_{2M}\theta}\right\} \\ &= \frac{-0.1668+0.1207+\ln 1.05}{3.81\times 1.05} \approx 0.0007. \end{aligned}$$

And so,

$$\begin{aligned} &\left(a_{1L}-\frac{c_{1M}M_2^{\beta}}{d_L}\right)\theta+\ln h_{1kL} = \left(2.25-\frac{0.005\times 2.2914^{0.2}}{0.539}\right)+\ln 2.25 \approx 3.0500 > 0, \\ &(a_{2M}-\xi)\theta = (0.1668-0.1207)\times 1 = 0.0461 < \ln h_{2kL} \\ &\qquad = \ln 1.05 \approx 0.0488 < a_{2L}\theta = 0.1666. \end{aligned}$$

All the conditions of Theorem 1 hold, then it follows from Theorem 1 that system (1.4) is permanent. Figures 1 and 2 support this results.



Fiugue 1: Dynamic behavior of prey x_1 in system (1.4) with the initial points $(1, 0.25)$, $(2.0.22)$ and $(3, 0.205)$.

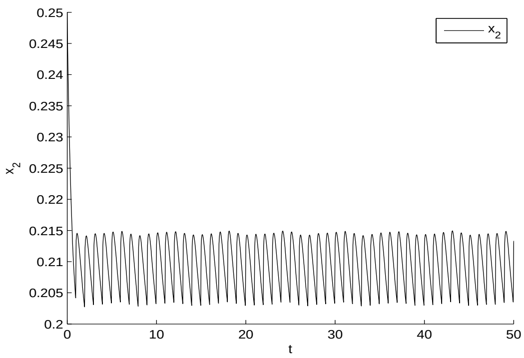


Figure 2: Dynamic behavior of predator x_2 in system (1.4) with the initial points $(1, 0.25)$, $(2, 0.22)$ and $(3, 0.205)$.

5 Discussion

Tripathi [4] considered the non-autonomous, density dependent predator-prey model with mutual interference and Crowley-Martin response function, the author probed into the system which admits a unique almost periodic solution. In this paper, we first generalize system (1.3) to the impulsive case, then under assumptions (H_1) and (H_2), by utilizing the theory of differential impulsive inequality and applying the analysis technique of He [14], the system is also permanent.

We would like to point out here that in this paper we do not obtain the results on the extinction, global attractivity of the system, the existence and uniqueness of positive periodic solutions, bifurcation and dynamical complexity and so forth. We leave this for future work.

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