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HYPERCYCLIC MULTIPLICATION COMPOSITION OPERATORS ON WEIGHTED BANACH SPACE*†

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Abstract

This paper characterizes some sufficient and necessary conditions for the hypercyclicity of multiples of composition operators on $H^\infty_{\log,0}$.

Keywords composition operators; hypercyclic; weighted banach space

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1 Introduction

Let $\mathbb{D}$ be an open unit disk in the complex plane $\mathbb{C}$. $H(\mathbb{D})$ denotes the space of all holomorphic functions on $\mathbb{D}$. $S(\mathbb{D})$ is the class of all holomorphic functions from the the open unit disk $\mathbb{D}$ in itself. Throughout this paper, log denotes the natural logarithm function, and ν denotes what we call a weight on $\mathbb{D}$; that is, ν is a bounded, continuous and strictly positive function defined on $\mathbb{D}$. The weighted Banach spaces of holomorphic functions $H^\infty_\nu(\mathbb{D})$, H^∞_ν for short, consists of all $f \in H(\mathbb{D})$ such that

$$\|f\|_\nu = \sup_{z\in\mathbb{D}} \nu(z)|f(z)| < \infty.$$

$H^\infty_{\nu,0}$ is the subspace of H^∞_ν consisting of $f \in H^\infty_\nu$ for which

$$\lim_{|z|\to 1} \nu(z)|f(z)| = 0.$$

Endowed with the weighted sup-norm $\|\cdot\|_\nu$, H^∞_ν and $H^\infty_{\nu,0}$ are both Banach spaces. As we all know the set of polynomials is dense in $H^\infty_{\nu,0}$, so that $H^\infty_{\nu,0}$ is a separable

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space. Particularly, let $\nu(z) = (1 - \log(1 - |z|^2))^\alpha$, $\alpha < 0$, and we will state and prove our main results acting on $H^\infty_{\log,0}$.

Each $\varphi \in S(\mathbb{D})$ induces a linear composition operator $C_\varphi : H(\mathbb{D}) \to H(\mathbb{D})$ defined by $C_\varphi f(z) = f(\varphi(z))$ with $f \in H(\mathbb{D})$ and $z \in \mathbb{D}$. A steadily increasing amount of attention has been paid to composition operator. We refer the reader to [3, 11, 12] for more details.

Let $\mathcal{L}(X)$ denote the space of linear and continuous operators on a separable, infinite dimensional Banach space X. Let $T \in \mathcal{L}(X)$. T is said to be hypercyclic if there is a vector $x \in X$ such that the orbit, defined as

$$Orb(T, x) := \{x, Tx, T^2x, T^3x, \cdots\},$$

is dense in X. Such a vector x is said to be a *hypercyclic vector* for the operator T.

The first example of a hypercyclic operator on a Banach space was given by Rolewicz [10] in 1969, which showed that if B is the unweighted unilateral backward shift on l^2, then λB is hypercyclic if and only if $|\lambda| > 1$. The notion of hypercyclic was first introduced into the field of linear dynamics in Kitai's doctoral dissertation [5], and since then this notion has been studied actively; see [1,4,6,7], and the references therein.

There are several forms to test the hypercyclicity of an operator, for examples [5, 9]. The following criterion is due to [4].

Theorem 1.1(Hypercyclicity Criterion) *Let T be an operator on a separable Banach space X. If there are dense subsets X_0, Y_0 in X, an increasing sequence $\{n_k\}$ of positive integers, and a map $S : X_0 \to X_0$ satisfying*

(i) $T^{n_k}x \to 0$ *for all* $x \in X_0$ *as* $k \to \infty$*;*

(ii) $S_{n_k}x \to 0$ *and* $T^{n_k}S_{n_k}y \to y$ *as* $k \to \infty$*, for all* $y \in Y_0$*.*

Then T is hypercyclic.

Let us recall a few preliminary facts and definitions on linear fractional transformations from [11]. We denote by $LFT(\widehat{\mathbb{C}})$ the group of linear fractional transformations, consisting of those bijections of the extended complex plane $(\widehat{\mathbb{C}}) = \mathbb{C} \cup \{\infty\}$ that are of the form $\varphi(z) = \frac{az+b}{cz+d}$, where $ad - bc \neq 0$. Two elements φ, ψ in $LFT(\widehat{\mathbb{C}})$ are said to be conjugate provided $\varphi = \sigma^{-1} \circ \psi \circ \sigma$ for some $\sigma \in LFT(\widehat{\mathbb{C}})$. The linear fractional composition operators are induced by members of the class $LFT(\mathbb{D}) = \{\varphi \in LFT(\widehat{\mathbb{C}}) : \varphi(\mathbb{D}) \subseteq \mathbb{D}\}$, and the invertible ones are induced by members of the subclass $Aut(\mathbb{D}) = \{\varphi \in LFT(\widehat{\mathbb{C}}) : \varphi(\mathbb{D}) = \mathbb{D}\}$ of automorphisms on the unit disc. The elements of $LFT(\mathbb{D})$ have the following fixed point configuration:

(a) Maps with interior fixed point. By the Schwarz lemma the interior fixed point is either attractive, or the map is an elliptic automorphism.

(b) Parabolic maps. Its fixed point is on $\partial\mathbb{D}$, and the derivative is 1 at the fixed point.

(c) Hyperbolic maps with attractive fixed point on $\partial\mathbb{D}$ and their repulsive fixed point outside of $\mathbb{D}$. Both fixed points are on $\partial\mathbb{D}$ if and only if the map is the automorphism of $\mathbb{D}$. In this case, the derivative is less than 1 at the attractive fixed point.

To obtain the behavior of φ^n, which is the iterates of analytic self-maps of $\mathbb{D}$, we give the following remarkable theorem.

Theorem 1.2(The Denjoy-Wolff Theorem) *If $\varphi : \mathbb{D} \to \mathbb{D}$ is an analytic map with no fixed point in $\mathbb{D}$. Then there exists a point $a \in \partial\mathbb{D}$ such that $\varphi^n \to a$ uniformly on compact subsets of $\mathbb{D}$.*

The point a is called the Denjoy-Wolff point of φ and φ has non-tangential limit at a.

Due to the classification of φ, the conditions for the hypercyclicity were discussed in [8].

Theorem 1.3 *Let v be a typical weight on $\mathbb{D}$ and $\varphi \in S(\mathbb{D})$. $C_\varphi : H_{v,0}^\infty \to H_{v,0}^\infty$ is continuous, then the following holds:*

(1) *If $\varphi \in Aut(\mathbb{D})$ fixes no point in $\mathbb{D}$, then C_φ is hypercyclic.*

(2) *If $\varphi \in LFT(\mathbb{D})$ is a hyperbolic non-automorphism, then C_φ is hypercyclic.*

(3) *Let $v(z) = (1-|z|)^{\frac{1}{2}}$. If $\varphi \in LFT(\mathbb{D})$ is a parabolic non-automorphism, then C_φ is not hypercyclic.*

There have been really good surveys on the topic of hypercyclicity; for example, [1, 4]. An operator T is called chaotic if it is hypercyclic and has a dense set of periodic points. One bounded operator T is called similar to another bounded operator S on X if there exists a bounded and invertible operator V on H such that $TV = VS$. And the similarity preserve hypercyclicity. A continuous linear operator T acting on a separable Banach space X is said to be mixing, if for any pair U, V of nonempty open subsets of X, there exists some $N \geq 0$ such that

$$T^n(U) \cap (V) \neq \emptyset, \quad \text{for all } n \geq N.$$

The paper is organized in the following manner: In Section 2, we provide some results which will be useful in proving our main theorem. In Section 3, we characterize some sufficient and necessary conditions for the hypercyclicity of multiples of composition operators on $H_{\log,0}^\infty$.

2 Notations and Lemmas

In this section, we will show some results which is useful in proving our main theorem.

Lemma 2.1[8] *Let m be any positive integer and $a \in \mathbb{C}$. If $|a| \geq 1$, then the subspace of all polynomials that vanish m times at a is dense in H_v^0.*

Lemma 2.2[4] *Let T be an operator on a complex Frechet space X. If $x \in X$ is such that $\{\lambda T^n x, \lambda \in \mathbb{C}, |\lambda| = 1,$ and $n \in \mathbb{N}_0\}$ is dense in X, then $Orb(x, \lambda T)$ is dense in X for each $\lambda \in \mathbb{C}$ with $|\lambda| = 1$. In particular, for any $\lambda \in \mathbb{C}$ with $|\lambda| = 1$, T and λT have the same hypercyclic vectors, that is, $HC(T) = HC(\lambda T)$.*

Lemma 2.3[4] *Let T be a hypercyclic operator on a complex Banach space X. Then the orbit of every $x^* \neq 0$ in X under the adjoint T^* is unbounded.*

Throughout the remainder of this paper, C denotes a positive constant, the exact value of which varies from one appearance to the next.

3 Hypercyclic Behaviour of Multiples of Composition Operators on H_V^0 Spaces

Theorem 3.1 *Let $\varphi \in LFT(\mathbb{D})$ be a hyperbolic automorphism and $\eta \in \partial\mathbb{D}$ be the Denjoy-Wolff point of φ. Then λC_φ is hypecyclic on H^0_{log} if and only if $\mu^{-\alpha} < |\lambda| < \mu^{\alpha}$.*

Proof Without loss of generality, we suppose that φ has fixed points 1 and -1. Meanwhile, 1 is the attractive one. We compute explicitly by employing again the change of variables

$$\sigma(z) = \frac{i(1+z)}{1-z}.$$

$\sigma(z)$ sends $\mathbb{D}$ onto the upper half plane, the fixed points 1 and -1 are changed into 0 and ∞ respectively. We can suppose

$$\varphi(z) = \frac{(1+\mu)z + 1 - \mu}{(1-\mu)z + 1 + \mu}, \quad 0 < \mu < 1.$$

The iterates are the following formulas

$$\varphi^n(z) = \frac{(1+\mu^n)z + 1 - \mu^n}{(1-\mu^n)z + 1 + \mu^n}, \quad \varphi^{-n}(z) = \frac{(1+\mu^n)z - 1 + \mu^n}{(-1+\mu^n)z + 1 + \mu^n}, \quad n \in \mathbb{N}.$$

So

$$1 - |\varphi^{-n}(z)|^2 = \frac{4\mu^n(1-|z|^2)}{|(\mu^n - 1)z + \mu^n + 1|^2}.$$

<u>Necessity</u> Now for any $f \in H^0_{\log}$, we have the following estimate

$$\begin{aligned}\|\lambda^n C_{\varphi^n} f\|_{\log} &= \sup_{z\in\mathbb{D}} |\lambda|^n (1 - \log(1-|z|^2))^\alpha |f(\varphi^n(z))| \\ &\leq \sup_{z\in\mathbb{D}} C|\lambda|^n \frac{(1-\log(1-|z|^2))^\alpha}{(1-\log(1-|\varphi^n(z)|^2))^\alpha} \\ &= C|\lambda|^n \sup_{z\in\overline{\varphi^n(\mathbb{D})}} \frac{(1-\log(1-|\varphi^{-n}(z)|^2))^\alpha}{(1-\log(1-|z|^2))^\alpha} \\ &\leq C|\lambda|^n \sup_{z\in\overline{\varphi^n(\mathbb{D})}} (1-|\varphi^{-n}(z)|^2)^\alpha\end{aligned}$$

$$= C|\lambda|^n \sup_{z\in\overline{\varphi^n(\mathbb{D})}} \Big(\frac{4\mu^n(1-|z|^2)}{|(\mu^n-1)z+\mu^n+1|^2}\Big)^\alpha$$
$$\le C|\lambda|^n\mu^{n\alpha}\sup_{z\in\mathbb{D}}\frac{|z-1|^{\alpha+m}|z+1|^\alpha}{(\mu^n+1-(1-\mu^n))^{2\alpha}} \le C|\lambda|^n\mu^{-n\alpha}. \tag{3.1}$$

When $|\lambda|\mu^{-1\alpha}\le 1$, $\lambda^n C_{\varphi^n}$ is bounded. If λC_φ is hypercylic, then $|\lambda|\mu^{-1\alpha}>1$. On the other hand, if λC_φ is hypercylic, $\lambda^{-1}C_{\varphi^{-1}}$ is hypercylic, too. So, we can obtain $|\lambda|\mu^{1\alpha}<1$.

Sufficiency Let X_0 be the set of all holomorphic functions on a neighborhood of $\overline{\mathbb{D}}$ that vanish m times at 1. Fix $f(z)=(z-1)^m g(z)\in X_0$, where $g(z)$ is a holomorphic function on a neighborhood of $\overline{\mathbb{D}}$. Note that

$$\begin{aligned}
\|\lambda^n C_{\varphi^n} f\|_{\log} &= \sup_{z\in\mathbb{D}} |\lambda|^n(1-\log(1-|z|^2))^\alpha|f(\varphi^n(z))|\\
&= \sup_{z\in\mathbb{D}} |\lambda|^n(1-\log(1-|z|^2))^\alpha|(\varphi^n(z)-1)^m g(\varphi^n(z))|\\
&= |\lambda|^n \sup_{z\in\overline{\varphi^n(\mathbb{D})}} |(1-\log(1-|\varphi^{-n}(z)|^2))^\alpha(z-1)^m g(z)|\\
&\le |\lambda|^n \sup_{z\in\mathbb{D}}|g(z)| \sup_{z\in\overline{\varphi^n(\mathbb{D})}} (1-|\varphi^{-n}(z)|^2)^\alpha|(z-1)^m|\\
&= C|\lambda|^n \sup_{z\in\overline{\varphi^n(\mathbb{D})}} \Big(\frac{4\mu^n(1-|z|^2)}{|(\mu^n-1)z+\mu^n+1|^2}\Big)^\alpha|(z-1)^m|\\
&\le C4^\alpha|\lambda|^n\mu^{n\alpha}\sup_{z\in\mathbb{D}}\frac{|z-1|^{\alpha+m}|z+1|^\alpha}{(\mu^n+1-(1-\mu^n))^{2\alpha}} \le C4^{2\alpha}|\lambda|^n\mu^{-n\alpha}.
\end{aligned} \tag{3.2}$$

Since $|\lambda|<\mu^\alpha$, for all $f\in X_0$,

$$\sum_{n=1}^{\infty}\|\lambda^n C_{\varphi^n} f\|_{\log}<+\infty. \tag{3.3}$$

Next, we consider the operator $S=\lambda^{-1}C_\varphi^{-1}=\lambda^{-1}C_{\varphi^{-1}}$ and let Y_0 be the set of holomorphic functions on a neighborhood of $\overline{\mathbb{D}}$ that vanish m times at -1. -1 is the attractive point of φ^{-1} with $(\varphi^{-1})'(-1)=\frac{1}{\varphi'(-1)}=\mu$. Because $|\lambda|>\mu^{-\alpha}$, as before, we can show that

$$\sum_{n=1}^{\infty}\|S^n f\|_{\log}<+\infty, \quad \text{for all } f\in Y_0. \tag{3.4}$$

Let $Z_0=X_0\cap Y_0$, by Lemma 2.1, Z_0 is dense in $H^0_{\log}$. Obviously, conditions (i) and (ii) of Theorem 1.1 hold for all $f\in Z_0$. Meanwhile, $\lambda C_\varphi S$ is identity and Z_0 is S-invariant, because φ^{-1} is conformal and fixed at the points 1 and -1. Thus λC_φ is hypercyclic. The proof is completed.

Theorem 3.2 *Let $\varphi\in LFT(\mathbb{D})$ be a hyperbolic non-automorphism and $\eta\in\partial\mathbb{D}$*

be the Denjoy-Wolff point of φ. *Then* λC_φ *is hypecyclic on* $H^0_{\log}$ *if and only if* $\mu^{-\alpha} < |\lambda|$.

Proof By Theorem 1.3 and Lemma 2.2, we suppose that φ has a fixed point 1. We use the change of variables

$$\sigma(z) = \frac{i(1+z)}{1-z}.$$

$\sigma(z)$ sends $\mathbb{D}$ onto the upper half plane, the fixed point 1 is changed into ∞ and the exterior fixed point is changed into a point p in the lower half plane. Upon conjugating with an appropriate affine map in the upper half plane, we may suppose that p is on the imaginary axis. Finally, for the unit disk $\mathbb{D}$, $\sigma(z)$ sends p to a negative number $a < -1$. Therefore, we may suppose that φ has following expression

$$\varphi(z) = \frac{(\mu a - 1)z + a(1-\mu)}{(\mu - 1)z + a + \mu}, \quad 0 < \mu < 1.$$

We conjugate one more time with $\frac{az-1}{a-z}$, which is an automorphism of $\mathbb{D}$ which fixes 1 and sends a to ∞. Therefore, we may suppose that

$$\varphi(z) = \mu z + 1 - \mu,$$

where $0 < \mu < 1$ with $\varphi'(1) = \mu$.

The iterates are the following formulas

$$\varphi^n(z) = \mu^n z + 1 - \mu^n.$$

If $f \in H^0_{\log}$, $\mu^{-\alpha} < |\lambda|$, we have the following estimate

$$\begin{aligned}
\|\lambda^n C_{\varphi^n} f\|_{\log} &= \sup_{z\in\mathbb{D}} |\lambda|^n (1-\log(1-|z|^2))^\alpha |f(\varphi^n(z))| \\
&\le \sup_{z\in\mathbb{D}} C|\lambda|^n \frac{(1-\log(1-|z|^2))^\alpha}{(1-\log(1-|\varphi^n(z)|^2))^\alpha} \\
&= C|\lambda|^n \sup_{z\in\mathbb{D}} \frac{1}{(1-\log(1-|z|^2))^\alpha} \\
&\le C|\lambda|^n \sup_{z\in\overline{\varphi^n(\mathbb{D})}} (1-|\varphi^{-n}(z)|^2)^\alpha \\
&= C|\lambda|^n \sup_{z\in\overline{\varphi^n(\mathbb{D})}} \Big(\frac{4\mu^n(1-|z|^2)}{|(\mu^n-1)z+\mu^n+1|^2}\Big)^\alpha \\
&\le C|\lambda|^n \mu^{n\alpha} \sup_{z\in\mathbb{D}} \frac{|z-1|^{\alpha+m}|z+1|^\alpha}{(\mu^n+1-(1-\mu^n))^{2\alpha}} \le C|\lambda|^n\mu^{-n\alpha}. \qquad (3.5)
\end{aligned}$$

When $|\lambda|\mu^{-1\alpha} \le 1$, $\lambda^n C_{\varphi^n}$ is bounded. If λC_φ is hypercylic, then $|\lambda|\mu^{-1\alpha} > 1$. On the other hand, if λC_φ is hypercylic, $\lambda^{-1} C_{\varphi_{-1}}$ is hypercylic, too. So, we can obtain $|\lambda|\mu^{1\alpha} < 1$.

Sufficiency Let X_0 be the set of all holomorphic functions on a neighborhood of $\overline{\mathbb{D}}$ that vanish m times at 1. Fix $f(z) = (z-1)^m g(z) \in X_0$, where $g(z)$ is a holomorphic function on a neighborhood of $\overline{\mathbb{D}}$.

$$\begin{aligned}
\|\lambda^n C_{\varphi^n} f\|_{\log} &= \sup_{z\in\mathbb{D}} |\lambda|^n (1-\log(1-|z|^2))^\alpha |f(\varphi^n(z))| \\
&= \sup_{z\in\mathbb{D}} |\lambda|^n (1-\log(1-|z|^2))^\alpha |(\varphi^n(z)-1)^m g(\varphi^n(z))| \\
&= |\lambda|^n \sup_{z\in\overline{\varphi^n(\mathbb{D})}} |(1-\log(1-|\varphi_{-n}(z)|^2))^\alpha (z-1)^m g(z)| \\
&\le |\lambda|^n \sup_{z\in\mathbb{D}} |g(z)| \sup_{z\in\overline{\varphi^n(\mathbb{D})}} (1-|\varphi^{-n}(z)|^2)^\alpha |(z-1)^m| \\
&= C|\lambda|^n \sup_{z\in\overline{\varphi^n(\mathbb{D})}} \Big(\frac{4\mu^n(1-|z|^2)}{|(\mu^n-1)z+\mu^n+1|^2}\Big)^\alpha |(z-1)^m| \\
&\le C4^\alpha |\lambda|^n \mu^{n\alpha} \sup_{z\in\mathbb{D}} \frac{|z-1|^{\alpha+m}|z+1|^\alpha}{(\mu^n+1-(1-\mu^n))^{2\alpha}} \\
&\le C4^{2\alpha}|\lambda|^n \mu^{-n\alpha}.
\end{aligned} \tag{3.6}$$

Since $|\lambda| < \mu^\alpha$, for all $f \in X_0$,

$$\sum_{n=1}^\infty \|\lambda^n C_{\varphi^n} f\|_{\log} < +\infty. \tag{3.7}$$

Next, we consider the operator $S = \lambda^{-1}C_\varphi^{-1} = \lambda^{-1}C_{\varphi^{-1}}$ and let Y_0 be the set of holomorphic functions on a neighborhood of $\overline{\mathbb{D}}$ that vanish m times at -1. -1 is the attractive point of φ^{-1} with $(\varphi^{-1})'(-1) = \frac{1}{\varphi'(-1)} = \mu$. Because $|\lambda| > \mu^{-\alpha}$, as before, we can show that

$$\sum_{n=1}^\infty \|S^n f\|_{\log} < +\infty, \quad \text{for all } f \in Y_0. \tag{3.8}$$

Let $Z_0 = X_0 \cap Y_0$, by Lemma 2.1, Z_0 is dense in $H^0_{\log}$. Obviously, (i) and (ii) of Theorem 1.1 hold for all $f \in Z_0$. Meanwhile, $\lambda C_\varphi S$ is identity and Z_0 is S-invariant, because φ^{-1} is conformal and fixed the points 1 and -1. Thus λC_φ satisfies Theorem 1.1. The proof is completed.

Theorem 3.3 *Let $\varphi \in LFT(\mathbb{D})$ be a parabolic non-automorphism of the unit disk. Then λC_φ is hypecyclic on $H^0_{\log}$ if and only if $|\lambda| = 1$.*

Proof Sufficiency If $|\lambda| = 1$, by Theorem 1.3, λC_φ is hypecyclic.

Necessity We suppose that φ has fixed point 1. We use the change of variables

$$\sigma(z) = \frac{i(1+z)}{1-z}.$$

$\sigma(z)$ sends $\mathbb{D}$ onto the upper half plane. Thus we may suppose that φ has following expression

$$\varphi(z)=\frac{(2-a)z+a}{-az+2+a},$$

where $a\neq 0$ and $\text{Re}a=0$.

The iterates are the following formula

$$\varphi^n(z)=\frac{(2-na)z+na}{-naz+2+na},\quad n\in\mathbb{N}.$$

Firstly, we suppose that $|\lambda|<1$. Choose $\delta\in(H^0_{\log})^*$ to be the point evaluation functional, that is $\delta(f)=f(0)$, then

$$|\langle(\lambda^nC_{\varphi^n})^*\delta,f\rangle|=|\lambda|^n|f(\varphi_n(0))|=|\lambda|^n\Big|f\Big(\frac{na}{2+na}\Big)\Big|\leq\frac{C|\lambda|^n}{\big(1-\log(1-|\frac{na}{2+na}|^2)\big)^\alpha}.\qquad(3.9)$$

Since $\text{Re}a=0$, it follows that $|2+na|^2-|na|^2=4$, therefore, By Lemma 2.3, λC_φ is not hypercyclic. Similarly, if $|\lambda|>1$, then $\lambda^{-1}C_{\varphi_{-1}}$ is not hypercyclic. So $|\lambda|=1$. This completes the proof.

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