

THE EFFECT OF REFUGE AND PROPORTIONAL HARVESTING FOR A PREDATOR-PREY SYSTEM WITH REACTION-DIFFUSION^{*†}

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Abstract

A diffusive predator-prey system with Holling-Tanner functional response and no-flux boundary condition is considered in this work. By using upper and lower solutions combined with iteration method, sufficient condition which ensures the global asymptotical stability of the unique positive equilibrium of the system is obtained. It is shown that the prey refuge and the proportional harvesting can influence the global asymptotical stability of unique positive equilibrium of the system, furthermore, they can change the position of the unique interior equilibrium and make species coexist more easily.

Keywords reaction-diffusion system; iteration method; global asymptotical stability; prey refuge; proportional harvesting

2000 Mathematics Subject Classification 35K57

1 Introduction

To accurately describe the real ecological interactions between some species such as lynx and hare, mite and spider mite, sparrow and sparrow hawk, etc. described by Wollkind et al. [1] and Tanner [2], Robert May proposed a Holling-Tanner predator-prey model [3], in which the author incorporated Holling's rate [4,5]. In [6], Hsu and Huang studied the following predator-prey system

$$\begin{cases} \frac{du}{dt} = ru\left(1 - \frac{u}{K}\right) - vp(u), \\ \frac{dv}{dt} = v\left[s\left(1 - \frac{hv}{u}\right)\right], \\ u(0) > 0, \quad v(0) > 0, \end{cases} \quad (1.1)$$

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where u and v are the populations of the prey and the predator respectively. K is the carrying capacity of the prey and r is the intrinsic growth rate in the absence of predation. s is the intrinsic growth rate of the predator and $p(u)$ is the functional response. The carrying capacity of the predator is proportional to the population size of the prey. By using Dulacs criterion and constructing Liapunov functions, they established the global stability of the positive locally asymptotically stable equilibrium of system (1.1). For more biological background of system (1.1), one could refer to [6-8] and the references cited therein.

Taking into account the distribution of the prey and predators in spatial location within a fixed bounded domain $\Omega \subset R^N$ ($N \leq 3$), Wonlyul Ko and Kimun Ryu [9] considered a Holling-Tanner predator-prey system with reaction-diffusion. In [10], Peng and Wang studied the following system

$$\begin{cases} \frac{\partial u}{\partial t} = d_1 \Delta u + u \left(a - u - \frac{v}{m+u} \right), & x \in \Omega, t > 0, \\ \frac{\partial v}{\partial t} = d_2 \Delta v + bv - \frac{v^2}{\gamma u}, & x \in \Omega, t > 0, \\ \frac{\partial u}{\partial \mathbf{n}} = \frac{\partial v}{\partial \mathbf{n}} = 0, & x \in \partial\Omega, t > 0, \\ u(x, 0) = u_0(x) > 0, \quad v(x, 0) = v_0(x) \geq (\neq) 0, & x \in \Omega, \end{cases} \quad (1.2)$$

where $u(x, t)$ and $v(x, t)$ are the species densities of the prey and predator respectively. The constants d_i ($i = 1, 2$) are the diffusion coefficients of prey and predator respectively. \mathbf{n} is the outward unit normal vector on the smooth boundary $\partial\Omega$. The initial datas $u_0(x)$ and $v_0(x)$ are continuous functions on $\overline{\Omega}$, the homogeneous Neumann boundary condition means that the system is self-contained and has no population flux across the boundary $\partial\Omega$. Obviously, as mentioned in [10], the above system has a unique coexisting positive equilibrium $(u, v) = (\bar{u}, \bar{v})$, where

$$\bar{u} = \frac{1}{2} \{ a - m - b\gamma + \sqrt{(a - m - b\gamma)^2 + 4am} \}, \quad \bar{v} = b\gamma\bar{u}. \quad (1.3)$$

They studied the stability of the positive constant solution of system (1.2) and obtained sufficient conditions for the global stability of the positive equilibrium by constructing a suitable Lyapunov function. For the ecological sense of system (1.2) we can refer to [10] and the references cited therein.

Recently, Chen and Shi [11] reconsidered the above system (1.2), and proved that if

$$m > b\gamma \quad (1.4)$$

holds, then the unique constant equilibrium of system (1.2) is globally asymptotically

stable, which shows that condition (1.4) is simpler and weaker than the condition obtained in [10].

As is well known, the existence of refuge has important effects on the coexistence of prey and predators, and research on the dynamic behaviors of predator-prey model with prey refuges has become a popular topic during the last decade [12-21]. If we extend model (1.2) by incorporating a refuge protecting su of the prey, where $s \in [0, 1]$ is a constant, $(1-s)u$ of the prey is available to the predator. Modifying model (1.2) accordingly becomes the following model:

$$\begin{cases} \frac{\partial u}{\partial t} = d_1 \Delta u + u(a-u) - \frac{(1-s)uv}{m+(1-s)u}, & x \in \Omega, t > 0, \\ \frac{\partial v}{\partial t} = d_2 \Delta v + bv - \frac{v^2}{\gamma(1-s)u}, & x \in \Omega, t > 0, \\ \frac{\partial u}{\partial \mathbf{n}} = \frac{\partial v}{\partial \mathbf{n}} = 0, & x \in \partial\Omega, t > 0, \\ u(x, 0) = u_0(x) > 0, \quad v(x, 0) = v_0(x) \geq (\neq) 0, & x \in \Omega. \end{cases} \quad (1.5)$$

Clearly, biological resources in the predator-prey model are most likely to be harvested and sold with the purpose of achieving the economic interest which motivates the introduction of harvesting in the predator-prey model. If we subject each population to a proportional harvesting effort specific to the population, the equations of model (1.2) become the following model

$$\begin{cases} \frac{\partial u}{\partial t} = d_1 \Delta u + u\left(a-u-\frac{v}{m+u}\right) - e_1 u, & x \in \Omega, t > 0, \\ \frac{\partial v}{\partial t} = d_2 \Delta v + bv - \frac{v^2}{\gamma u} - e_2 v, & x \in \Omega, t > 0, \\ \frac{\partial u}{\partial \mathbf{n}} = \frac{\partial v}{\partial \mathbf{n}} = 0, & x \in \partial\Omega, t > 0, \\ u(x, 0) = u_0(x) > 0, \quad v(x, 0) = v_0(x) \geq (\neq) 0, & x \in \Omega, \end{cases} \quad (1.6)$$

where constants e_i ($i = 1, 2$) are harvesting efforts on respective populations.

To the best of the author's knowledge, seldom did scholars consider effect of the refuge and the proportional harvesting. Specially, to this day, still no scholars investigate whether the refuge and the proportional harvesting can influence the global stability of the positive equilibrium of system (1.2) or not.

Motivated by the above question, the main concern of this paper is to study the effect of the refuge and the proportional harvesting. More concretely, in the following section we will investigate that the unique interior equilibrium of system (1.5) is globally asymptotically stable under some simple conditions by using upper

and lower solutions method, and we will also discuss the influence of the prey refuge. Secondly, we will discuss the influence of the proportional harvesting in Section 3. Finally, we end this paper with a brief conclusion in Section 4.

2 The Influence of Prey Refuge

It is easy to verify that system (1.5) has a unique positive equilibrium (u_*, v_*) , where

$$\begin{aligned} u_* &= \frac{a(1-s) - m - (1-s)^2 b\gamma + \sqrt{[a(1-s) - m - (1-s)^2 b\gamma]^2 + 4am(1-s)}}{2(1-s)}, \\ v_* &= b\gamma(1-s)u_*. \end{aligned} \quad (2.1)$$

Now, we give the result of the global stability of (u_*, v_*) for system (1.5), which implies the prey and predators are spatially homogeneously distributed as the time converges to infinity.

Theorem 2.1 *Assume that*

$$m > b\gamma(1-s)^2, \quad (2.2)$$

then the positive equilibrium (u_, v_*) is globally attractive for system (1.5), that is, for any initial values $u_0(x) > 0$, $v_0(x) > 0$,*

$$\lim_{t \rightarrow +\infty} u(x, t) = u_*, \quad \lim_{t \rightarrow +\infty} v(x, t) = v_*,$$

uniformly for $x \in \bar{\Omega}$.

Proof It is well known that if $c > 0$ and $\omega(x, t)$ satisfies the equation

$$\begin{cases} \frac{\partial \omega}{\partial t} = D\Delta\omega + \omega(c - \omega), & x \in \Omega, \ t > 0, \\ \frac{\partial \omega(t, x)}{\partial \nu} = 0, & x \in \partial\Omega, \ t > 0, \\ \omega(x, 0) \geq (\neq) 0, & x \in \Omega. \end{cases} \quad (2.3)$$

then $\omega(t, x) \rightarrow c$ uniformly for $x \in \bar{\Omega}$ as $t \rightarrow +\infty$ (see [9]).

Since (2.2) holds, we can choose an ε_0 satisfying

$$0 < \varepsilon_0 < \frac{[m - b\gamma(1-s)^2]a}{b\gamma(1-s)^2 + (1-s) + m + m[b\gamma(1-s)]^{-1}}. \quad (2.4)$$

From the first equation of system (1.5), we can easily obtain that

$$\frac{\partial u}{\partial t} = d_1 \Delta u + u(a - u) - \frac{(1-s)uv}{m + (1-s)u} \leq d_1 \Delta u + u(a - u), \quad (2.5)$$

then from comparison principle of parabolic equations, there exists a $t_1 > 0$ such that for any $t > t_1$, $u(x, t) \leq \bar{c}_1$, where $\bar{c}_1 = a + \varepsilon_0$.

Therefore, from the second equation of system (1.5), we have

$$\frac{\partial v}{\partial t} = d_2 \Delta v + v \left(b - \frac{v}{\gamma(1-s)u} \right) \leq d_2 \Delta v + v \left(b - \frac{v}{\gamma(1-s)(a + \varepsilon_0)} \right), \quad (2.6)$$

for $t > t_1$. Hence there exists a $t_2 > t_1$ such that for any $t > t_2$, $v(x, t) \leq \bar{c}_2$, where $\bar{c}_2 = b\gamma(1-s)(a + \varepsilon_0) + \varepsilon_0$. Again this implies

$$\begin{aligned} \frac{\partial u}{\partial t} &= d_1 \Delta u + u \left(a - u - \frac{(1-s)v}{m + (1-s)u} \right) \\ &\geq d_1 \Delta u + u \left(a - u - \frac{(1-s)[b\gamma(1-s)(a + \varepsilon_0) + \varepsilon_0]}{m} \right) \\ &= d_1 \Delta u + u \left(a - \frac{(1-s)[b\gamma(1-s)(a + \varepsilon_0) + \varepsilon_0]}{m} - u \right), \end{aligned} \quad (2.7)$$

for $t > t_2$. Since (2.2) holds, for ε_0 chosen as in (2.4), there are

$$a - \frac{(1-s)[b\gamma(1-s)(a + \varepsilon_0) + \varepsilon_0]}{m} > 0 \quad (2.8)$$

and

$$a - \frac{(1-s)[b\gamma(1-s)(a + \varepsilon_0) + \varepsilon_0]}{m} - \varepsilon_0 > 0. \quad (2.9)$$

Hence there exists a $t_3 > t_2$ such that for any $t > t_3$, $u(x, t) \geq \underline{c}_1$, where

$$\underline{c}_1 = a - \frac{(1-s)[b\gamma(1-s)(a + \varepsilon_0) + \varepsilon_0]}{m} - \varepsilon_0.$$

Finally we apply the lower bound of u to the second equation of system (1.5), and have

$$\begin{aligned} \frac{\partial v}{\partial t} &= d_2 \Delta v + v \left(b - \frac{v}{\gamma(1-s)u} \right) \\ &\geq d_2 \Delta v + v \left(b - \frac{v}{\gamma(1-s) \left[a - \frac{(1-s)[b\gamma(1-s)(a + \varepsilon_0) + \varepsilon_0]}{m} - \varepsilon_0 \right]} \right), \end{aligned} \quad (2.10)$$

for $t > t_3$. Since for ε_0 chosen as in (2.4),

$$b\gamma(1-s) \left[a - \frac{(1-s)[b\gamma(1-s)(a + \varepsilon_0) + \varepsilon_0]}{m} - \varepsilon_0 \right] - \varepsilon_0 > 0. \quad (2.11)$$

Then there exists a $t_4 > t_3$ such that for any $t > t_4$, $v(x, t) \geq \underline{c}_2$, where

$$\underline{c}_2 = b\gamma(1-s) \left[a - \frac{(1-s)[b\gamma(1-s)(a + \varepsilon_0) + \varepsilon_0]}{m} - \varepsilon_0 \right] - \varepsilon_0 > 0.$$

Therefore, for $t > t_4$ we obtain that

$$\underline{c}_1 \leq u(x, t) \leq \bar{c}_1, \quad \underline{c}_2 \leq v(x, t) \leq \bar{c}_2, \quad (2.12)$$

and $\underline{c}_1, \underline{c}_2, \bar{c}_1, \bar{c}_2$ satisfy

$$\begin{aligned} a - \bar{c}_1 - \frac{(1-s)\underline{c}_2}{m + (1-s)\bar{c}_1} &\leq 0 \leq a - \underline{c}_1 - \frac{(1-s)\bar{c}_2}{m + (1-s)\underline{c}_1}, \\ b - \frac{\bar{c}_2}{\gamma(1-s)\bar{c}_1} &\leq 0 \leq b - \frac{\underline{c}_2}{\gamma(1-s)\underline{c}_1}. \end{aligned} \quad (2.13)$$

The inequalities (2.13) show that (\bar{c}_1, \bar{c}_2) and $(\underline{c}_1, \underline{c}_2)$ are a pair of coupled upper and lower solutions of system (1.5) as in the definition in [22,23], as the nonlinearities in (1.5) are mixed quasimonotone. It is clear that there exists a $K > 0$ such that for any $(\underline{c}_1, \underline{c}_2) \leq (u_1, v_1), (u_2, v_2) \leq (\bar{c}_1, \bar{c}_2)$,

$$\begin{aligned} \left| \left[u_1(a - u_1) - \frac{(1-s)u_1v_1}{m + (1-s)u_1} \right] - \left[u_2(a - u_2) - \frac{(1-s)u_2v_2}{m + (1-s)u_2} \right] \right| &\leq K(|u_1 - u_2| + |v_1 - v_2|), \\ \left| v_1 \left(b - \frac{v_1}{\gamma(1-s)u_1} \right) - v_2 \left(b - \frac{v_2}{\gamma(1-s)u_2} \right) \right| &\leq K(|u_1 - u_2| + |v_1 - v_2|). \end{aligned} \quad (2.14)$$

We define two iteration sequences $(\bar{c}_1^{(n)}, \bar{c}_2^{(n)})$ and $(\underline{c}_1^{(n)}, \underline{c}_2^{(n)})$ as follows: For $n \geq 1$,

$$\begin{aligned} \bar{c}_1^{(n)} &= \bar{c}_1^{(n-1)} + \frac{1}{K} \bar{c}_1^{(n-1)} \left(a - \bar{c}_1^{(n-1)} - \frac{(1-s)\bar{c}_2^{(n-1)}}{m + (1-s)\bar{c}_1^{(n-1)}} \right), \\ \bar{c}_2^{(n)} &= \bar{c}_2^{(n-1)} + \frac{1}{K} \bar{c}_2^{(n-1)} \left(b - \frac{\bar{c}_2^{(n-1)}}{\gamma(1-s)\bar{c}_1^{(n-1)}} \right), \\ \underline{c}_1^{(n)} &= \underline{c}_1^{(n-1)} + \frac{1}{K} \underline{c}_1^{(n-1)} \left(a - \underline{c}_1^{(n-1)} - \frac{(1-s)\underline{c}_2^{(n-1)}}{m + (1-s)\underline{c}_1^{(n-1)}} \right), \\ \underline{c}_2^{(n)} &= \underline{c}_2^{(n-1)} + \frac{1}{K} \underline{c}_2^{(n-1)} \left(b - \frac{\underline{c}_2^{(n-1)}}{\gamma(1-s)\underline{c}_1^{(n-1)}} \right), \end{aligned} \quad (2.15)$$

where $(\bar{c}_1^{(0)}, \bar{c}_2^{(0)}) = (\bar{c}_1, \bar{c}_2)$ and $(\underline{c}_1^{(0)}, \underline{c}_2^{(0)}) = (\underline{c}_1, \underline{c}_2)$. Then for $n \geq 1$,

$$(\underline{c}_1, \underline{c}_2) \leq (\underline{c}_1^{(n)}, \underline{c}_2^{(n)}) \leq (\underline{c}_1^{(n+1)}, \underline{c}_2^{(n+1)}) \leq (\bar{c}_1^{(n+1)}, \bar{c}_2^{(n+1)}) \leq (\bar{c}_1^{(n)}, \bar{c}_2^{(n)}) \leq (\bar{c}_1, \bar{c}_2),$$

and there exist $(\tilde{c}_1, \tilde{c}_2)$ and $(\check{c}_1, \check{c}_2)$ such that

$$(\underline{c}_1, \underline{c}_2) \leq (\check{c}_1, \check{c}_2) \leq (\tilde{c}_1, \tilde{c}_2) \leq (\bar{c}_1, \bar{c}_2).$$

So

$$\lim_{n \rightarrow +\infty} \bar{c}_1^{(n)} = \tilde{c}_1, \quad \lim_{n \rightarrow +\infty} \bar{c}_2^{(n)} = \tilde{c}_2, \quad \lim_{n \rightarrow +\infty} \underline{c}_1^{(n)} = \check{c}_1, \quad \lim_{n \rightarrow +\infty} \underline{c}_2^{(n)} = \check{c}_2. \quad (2.16)$$

Hence, from (2.15) and (2.16) we have

$$\begin{aligned} a - \tilde{c}_1 - \frac{(1-s)\check{c}_2}{m + (1-s)\tilde{c}_1} &= 0, & a - \check{c}_1 - \frac{(1-s)\tilde{c}_2}{m + (1-s)\check{c}_1} &= 0, \\ b - \frac{\tilde{c}_2}{\gamma(1-s)\tilde{c}_1} &= 0, & b - \frac{\check{c}_2}{\gamma(1-s)\check{c}_1} &= 0. \end{aligned} \quad (2.17)$$

Simplifying (2.17) we obtain

$$\begin{aligned} (a - \tilde{c}_1)[m + (1-s)\tilde{c}_1] &= b\gamma(1-s)^2\check{c}_1, \\ (a - \check{c}_1)[m + (1-s)\check{c}_1] &= b\gamma(1-s)^2\tilde{c}_1. \end{aligned} \quad (2.18)$$

Subtracting the first equation of (2.18) from the second equation of (2.18), we can obtain that

$$(\tilde{c}_1 - \check{c}_1)[m - a(1-s) - b\gamma(1-s)^2 + (1-s)(\tilde{c}_1 + \check{c}_1)] = 0. \quad (2.19)$$

If we assume that $\tilde{c}_1 \neq \check{c}_1$, then

$$\tilde{c}_1 + \check{c}_1 = a - \frac{m - b\gamma(1-s)^2}{1-s}. \quad (2.20)$$

Substituting equation (2.20) into (2.18), we have

$$\begin{aligned} (a - \tilde{c}_1)[m + (1-s)\tilde{c}_1] &= b\gamma(1-s)^2 \left[a - \frac{m - b\gamma(1-s)^2}{1-s} - \tilde{c}_1 \right], \\ (a - \check{c}_1)[m + (1-s)\check{c}_1] &= b\gamma(1-s)^2 \left[a - \frac{m - b\gamma(1-s)^2}{1-s} - \check{c}_1 \right]. \end{aligned} \quad (2.21)$$

Hence the following equation

$$(a - y)[m + (1-s)y] = b\gamma(1-s)^2 \left[a - \frac{m - b\gamma(1-s)^2}{1-s} - y \right] \quad (2.22)$$

has two positive roots \tilde{c}_1, \check{c}_1 . From (2.2) and (2.20), we have

$$0 < a - \frac{m - b\gamma(1-s)^2}{1-s} < a. \quad (2.23)$$

(2.22) can be written as follows

$$(1-s)y^2 + [m - a(1-s) - (1-s)^2 b\gamma]y + (1-s)^2 b\gamma \left[a - \frac{m - b\gamma(1-s)^2}{1-s} \right] - am = 0. \quad (2.24)$$

From (2.2) and (2.23), it follows that

$$(1-s)^2 b\gamma \left[a - \frac{m - b\gamma(1-s)^2}{1-s} \right] - am < (1-s)^2 b\gamma a - am = a(b\gamma(1-s)^2 - m) < 0. \quad (2.25)$$

From (2.25), we can easily obtain that (2.22) do not exist two positive roots. Hence $\tilde{c}_1 = \check{c}_1$, and consequently, $\tilde{c}_2 = \check{c}_2$. Then from the results in [17,18], the solution $(u(x, t), v(x, t))$ of system (1.5) satisfies

$$\lim_{t \rightarrow +\infty} u(x, t) = u_*, \quad \lim_{t \rightarrow +\infty} v(x, t) = v_*,$$

uniformly for $x \in \overline{\Omega}$. So the constant equilibrium (u_*, v_*) is globally asymptotically stable for system (1.5).

This completes the proof of Theorem 2.1.

Remark 2.1 If we consider system (1.5) with $s = 0$, Theorem 2.1 is reduced to the main results of Chen and Shi [11], so our results generalize the main results in [11].

Remark 2.2 In [11], Chen and Shi considered system (1.2). Under the assumption $m > b\gamma$, the unique positive equilibrium of system (1.2) is globally asymptotically stable. In Theorem 2.1, we obtain that if $m > b\gamma(1-s)^2$ holds, then the unique positive equilibrium of system (1.5) is globally asymptotically stable. One can show that the parameter region given by $m > b\gamma(1-s)^2$ is contained in the set given by $m > b\gamma$. That is, if $m > b\gamma$ holds, then $m > b\gamma(1-s)^2$ also holds. It is shown that the prey refuge has influence on the global asymptotical stability of unique positive equilibrium of system (1.5), furthermore, they can change the position of the unique interior equilibrium and make species coexist more easily.

Remark 2.3 From condition (2.2) of Theorem 2.1, we can easily obtain that if the saturation constant and refuge constant of prey are large enough, the intrinsic growth rate of predator and the conversion factor of prey into predator are relatively small, then the positive equilibrium (u_*, v_*) is globally attractive for system (1.5). From the viewpoint of biology, this implies that the prey and predator will be spatially homogeneously distributed as the time converges to infinity, no matter what their diffusion coefficients are.

Remark 2.4 We will give numerical simulation to show the feasibility of our results. In system (1.5), set $a = 1$; $m = 2$; $b = 1$; $\gamma = 1$; $s = 0.5$. By computation, one has

$$m > b\gamma(1-s)^2,$$

then condition (2.2) of Theorem 2.1 holds. Figure 1 shows the dynamics behavior of system (1.5).

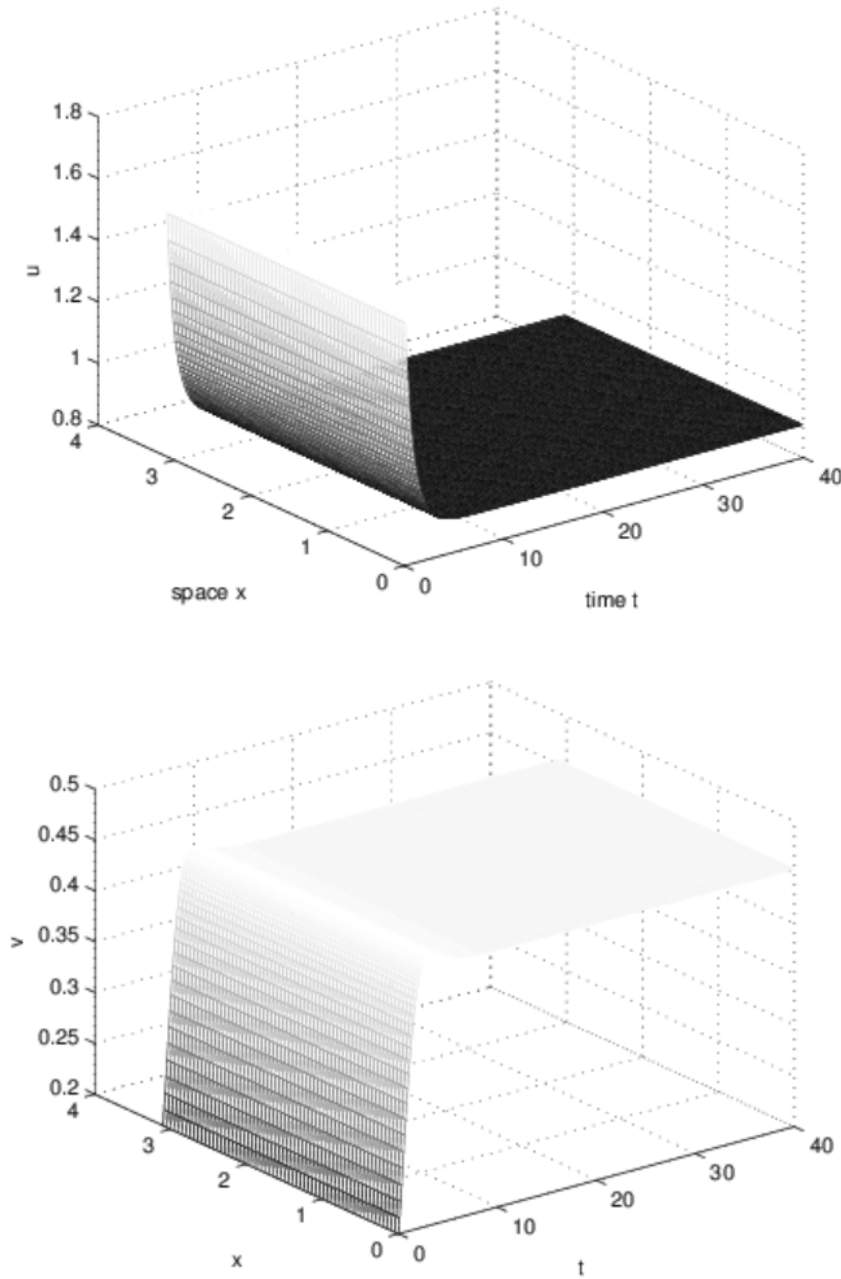


Figure 1: Dynamics behavior of system (1.5) with $a = 1$; $m = 2$; $b = 1$; $\gamma = 1$; $s = 0.5$.

3 The Influence of Proportional Harvesting

By simple computation, system (1.6) admits a unique positive equilibrium $(u, v) = (u^*, v^*)$, if

$$a > e_1, \quad b > e_2, \quad (3.1)$$

holds, where

$$\begin{aligned} u^* &= \frac{1}{2} \{ a - e_1 - m - (b - e_2)\gamma + \sqrt{[a - e_1 - m - (b - e_2)\gamma]^2 + 4(a - e_1)m} \}, \\ v^* &= (b - e_2)\gamma u^*. \end{aligned} \quad (3.2)$$

Similar to the proofs of Theorem 2.1 in Section 2, by using upper and lower solutions method, we can easily obtain the following theorem.

Theorem 3.1 *If (3.1) and the following inequality*

$$m > (b - e_2)\gamma, \quad (3.3)$$

holds, then the unique positive equilibrium of system (1.6) is globally asymptotically stable.

Remark 3.1 From Theorem 3.1, we obtain that under assumption (3.1), if $m > (b - e_2)\gamma$ holds, then the unique positive equilibrium of system (1.6) is globally asymptotically stable. One can also show that the parameter region given by $m > (b - e_2)\gamma$ is contained in the set given by $m > b\gamma$. It is also shown that under some assumption, the proportional harvesting has influence on the global asymptotical stability of unique positive equilibrium of system (1.6), furthermore, they can change the position of the unique interior equilibrium and make species coexist more easily. As we know, biological resources in the prey-predator system are most likely to be harvested and sold with the purpose of achieving the economic interest. From Theorem 3.1, if we choose and control the harvesting efforts e_i , $i = 1, 2$, appropriately, we can not only harvest the biological resources and achieve the economic interest, but also protect the biological resources.

Remark 3.2 We will give numerical simulation to show the feasibility of our results. In system (1.6), set $a = 1$; $m = 2$; $b = 1$; $\gamma = 1$; $e_1 = 0.5$; $e_2 = 0.5$. By computation, one can easily obtain that conditions (3.1) and (3.3) of Theorem 3.1 hold. Figure 2 shows the dynamics behavior of system (1.6).

4 Conclusion and Discussion

In this paper, by using upper and lower solutions combined with the iteration method, we obtain the global asymptotical stability of systems (1.5) and (1.6). It is shown that the prey refuge and the proportional harvesting have influence on the global asymptotical stability of unique positive equilibrium of system (1.2), furthermore, they can change the position of the unique interior equilibrium and make species coexist more easily. Indeed, similar to proofs of Theorem 2.1 in Section

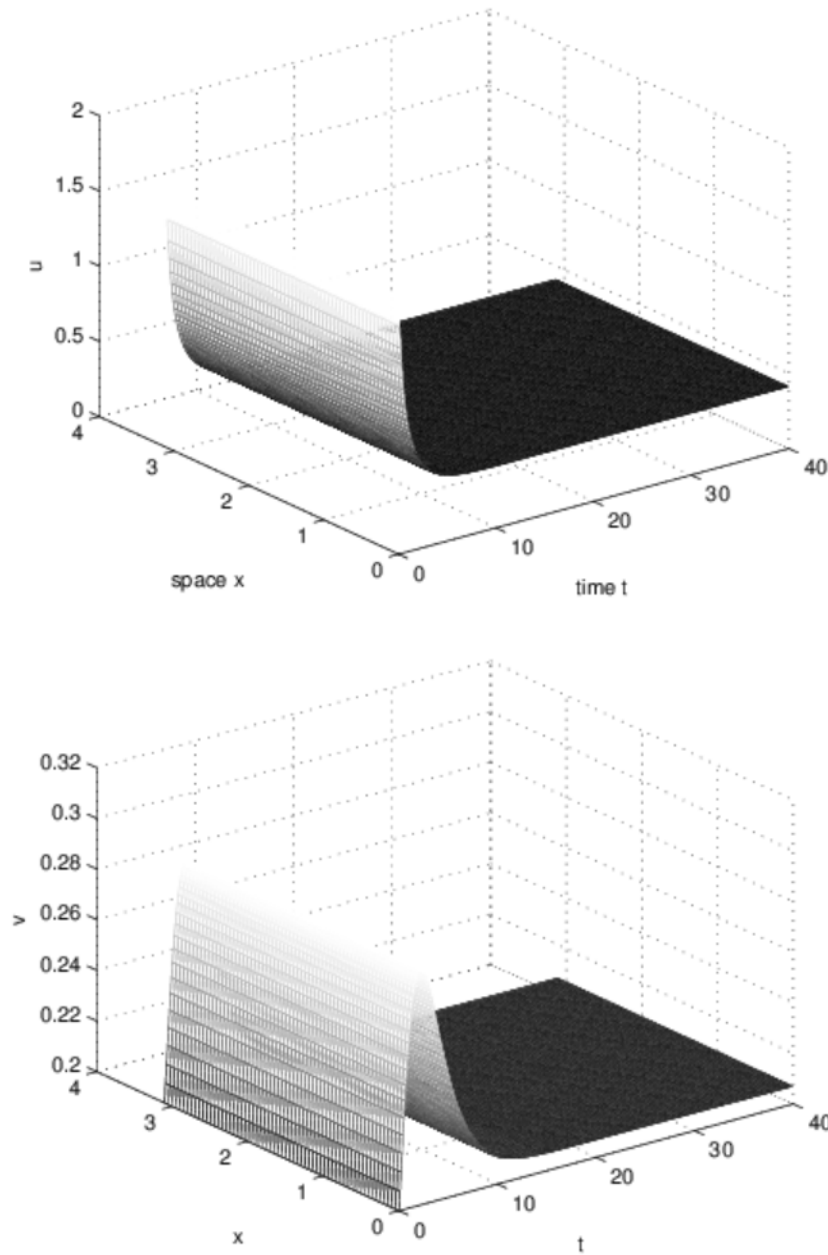


Figure 2: Dynamics behavior of system (1.6) with $a = 1$; $m = 2$; $b = 1$; $\gamma = 1$; $e_1 = 0.5$; $e_2 = 0.5$.

2, we can easily obtain that the unique positive equilibrium of the following diffusive predator-prey system with prey refuge and proportional harvesting

$$\begin{cases} \frac{\partial u}{\partial t} = d_1 \Delta u + u(a - u) - \frac{(1-s)uv}{m + (1-s)u} - e_1 u, & x \in \Omega, t > 0, \\ \frac{\partial v}{\partial t} = d_2 \Delta v + bv - \frac{v^2}{\gamma(1-s)u} - e_2 u, & x \in \Omega, t > 0, \\ \frac{\partial u}{\partial \mathbf{n}} = \frac{\partial v}{\partial \mathbf{n}} = 0, & x \in \partial\Omega, t > 0, \\ u(x, 0) = u_0(x) > 0, v(x, 0) = v_0(x) \geq (\neq) 0, & x \in \Omega \end{cases} \quad (4.1)$$

is also globally asymptotically stable, if

$$a > e_1, \quad b > e_2, \quad (4.2)$$

and

$$m > (b - e_2)\gamma(1 - s)^2, \quad (4.3)$$

hold. Therefore, if we choose and control the prey refuge parameter and the harvesting efforts appropriately, we can not only harvest the biological resources and achieve the economic interest, but also protect the biological resources.

References

- [1] D.J. Wollkind, J.B. Collings, J.A. Logan, Metastability in a temperature-dependent model system for predator-prey mite outbreak interactions on fruit flies, *Bull. Math. Biol.*, **50**(1988),379-409.
- [2] J.T. Tanner, The stability and the intrinsic growth rates of prey and predator populations, *Ecology*, **56**(1975),855-867.
- [3] E. Saez, E. Gonzalez-Olivares, Dynamics of a predator-prey model, *SIAM J. Appl. Math.*, **59**(1999),1867-1878.
- [4] M.P. Hassell, The Dynamics of Arthropod Predator-Prey Systems, Princeton University Press, Princeton, NJ, 1978.
- [5] C.S. Holling, The functional response of invertebrate predators to prey density, *Mem. Entomol. Soc. Can.*, **45**(1965),3-60.
- [6] S.B. Hsu, T.W. Huang, Global stability for a class of predator-prey systems, *SIAM J. Appl. Math.*, **55**(1995),763-783.
- [7] M.C. Montano, B.Lisena, Diffusive Holling-Tanner predator-prey models in periodic environments, *Applied Mathematics Letters*, **87**(2019),42-49.
- [8] W.S. Yang, Dynamical behaviors of a diffusive predator-prey model with Beddington-DeAngelis functional response and disease in the prey, *International Journal of Biomathematics*, **10**:8(2017),1750119.
- [9] W. Ko, K. Ryu, Non-constant positive steady-states of a diffusive predator-prey system in homogeneous environment, *J. Math. Anal. Appl.*, **327**(2007),539-549.
- [10] R. Peng, M.X. Wang, Global stability of the equilibrium of a diffusive Holling-Tanner prey-predator model, *Appl. Math. Lett.*, **20**(2007),664-670.

- [11] S.S. Chen, J.P. Shi, Global stability in a diffusive Holling-Tanner predator-prey model, *Applied Mathematics Letters*, **25**(2012),614-618.
- [12] T. Kumar Kar, Modelling and analysis of a harvested prey-predator system incorporating a prey refuge, *J. Comput. Appl. Math.*, **185**(2006),19-33.
- [13] W. Ko, K. Ryu, Qualitative analysis of a predator-prey model with Holling type II functional response incorporating a prey refuge, *J. Differential Equations*, **231**(2006),534-550.
- [14] Y. Huang, F. Chen, Z. Li, Stability analysis of a prey-predator model with Holling type III response function incorporating a prey refuge, *Appl. Math. Comput.*, **182**(2006),672-683.
- [15] L.L. Ji, C.Q. Wu, Qualitative analysis of a predator-prey model with constant-rate prey harvesting incorporating a constant prey refuge, *Nonlinear Anal.: Real World Appl.*, **11**:4(2010),2285-2295.
- [16] X.N. Guan, W.M. Wang, Y.L. Cai, Spatiotemporal dynamics of a Leslie-Gower predator-prey model incorporating a prey refuge, *Nonlinear Anal.: Real World Appl.*, **12**:4(2011),2385-2395.
- [17] Z.H. Ma, The research of predator-prey models incorporating prey refuges, Ph.D. Thesis, Lanzhou University, 2010.
- [18] F.D. Chen, Z.Z. Ma, H.Y. Zhang, Global asymptotical stability of the positive equilibrium of the Lotka-Volterra prey-predator model incorporating a constant number of prey refuges, *Nonlinear Anal.: Real World Appl.*, **13**(2012),2790-2793.
- [19] X.L. Zou, J.L. Lv, Y.P. Wu, A note on a stochastic Holling-II predator-prey model with a prey refuge, *Journal of the Franklin Institute*, **357**:7(2020),4486-4502.
- [20] U. Ufuktepe, Stability analysis of a prey refuge predator-prey model with Allee effects, *Journal of Biosciences*, **44**:4(2019),85.
- [21] P. Yang, Hopf bifurcation of an age-structured prey-predator model with Holling type II functional response incorporating a prey refuge, *Nonlinear Analysis: Real World Applications*, **49**(2019),368-385.
- [22] C.V. Pao, On nonlinear reaction-diffusion systems, *J. Math. Anal. Appl.*, **87**:1(1982),165-198.
- [23] C.V. Pao, Nonlinear Parabolic and Elliptic Equations, Plenum Publishing Corporation, 1992.

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